Step-by-Step Solution to Selected Problems in SIGNALS & SYSTEMS (Question-only Version)

Hamid Saeedi
University of Doha for Science and Technology

Hossein Pishro-Nik
University of Massachusetts Amherst

Published by Kappa Research, LLC.

Printed in the United States of America

ISBN: 978-0-9906372-3-3

Contents

Preface

Courses on continuous and discrete-time signals and systems are a fundamental part of most Electrical Engineering programs. As a result, there are numerous textbooks on this subject, each offering a variety of examples and unsolved problems for assignments. Additionally, there are books available that provide solutions to these problems or similar ones. However, after evaluating many of these resources, we identified a gap: a lack of books that guide students step-by-step through the solutions. Even when such resources exist, the difficulty level of the problems often fails to adequately prepare students for a rigorous final exam on signals and systems. This realization motivated us to co-author this book. In some similar books, solutions frequently skip steps, assuming students are already familiar with the missing information. In this book, we strive to avoid such assumptions and provide detailed, step-by-step solutions.

The book consists of 10 chapters with over 200 problems, each containing multiple questions on related topics. The chapters are organized to align with the sequence found in most signals and systems textbooks, particularly following the structure of the seminal work by Oppenheim & Willsky[1]. At the beginning of each chapter, we offer a concise yet focused review of the topic to minimize the need for students to refer back to the textbook. We primarily adhere to the notations and terminology used in Oppenheim & Willsky's book.

The preparation of this book would not have been possible without the invaluable assistance of two of our students, Mr. Mohamad Khas and Mr. Mohamad Mir Ahmadi. They were involved in every step of the process, and we relied on their perspectives as students to ensure that the solutions are as accessible as possible for those using this book. The second author has previously published a well-received book on probability and stochastic processes. We hope this book will be similarly successful and help students enhance their practical skills in this important subject.

[1]Oppenheim, A. V., and Willsky, A. S., with S. H. Nawab, Signals and Systems, Second Edition, Upper Saddle River, NJ: Prentice Hall, Inc., 1997

Chapter 1

Continuous-time and Discrete-time Signals

Problem 1. Determine which of the following signals are periodic. If a signal is periodic, determine its fundamental period.

(a) $y(t) = \sin(\frac{5\pi}{4}t)$

(b) $y(t) = e^{j(t+\frac{\pi}{3})}$

(c) $y(t) = \cos(\frac{3}{5}t) + |\sin(\frac{5}{3}t)|$

Problem 2. A continuous-time signal $x(t)$ is shown in Fig. 1.1. Find the fundamental period of $y(t) = x(\cos^2(t))$.

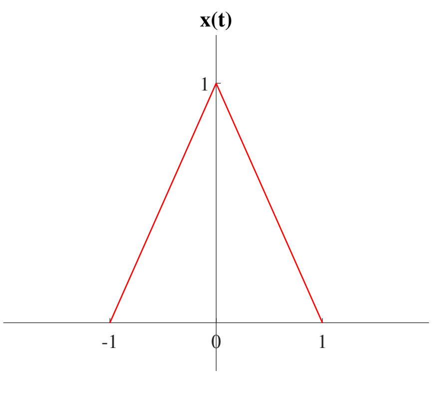


Figure 1.1: Problem 2.

Problem 3. Sketch the following signals and determine and sketch their even and odd components (Hint : $\prod(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$).

(a) $x(t) = (\frac{t+1}{2})\prod(\frac{t}{2}) + (2-t)\prod(t-\frac{3}{2})$

(b) $x(t) = 2e^{2t}u(-t) - e^{-2t}u(t)$

(c) $x(t) = \prod(\frac{t-6}{6})$

Problem 4. Let $x_o(t)$ be the odd component of the signal shown in Fig. 1.2. Compute the following integral:

$$\int_0^\infty x_o(t)dt \ = ?$$

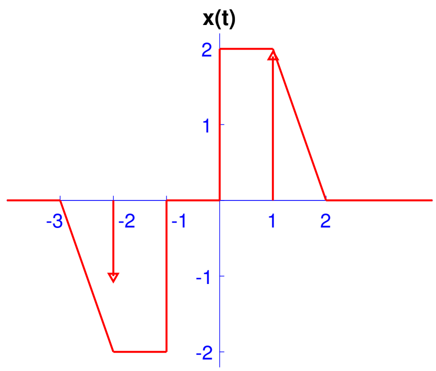


Figure 1.2: Problem 4.

Problem 5. A continuous-time signal $x(t)$ is shown in Fig. 1.3. Obtain the second derivative of this signal.

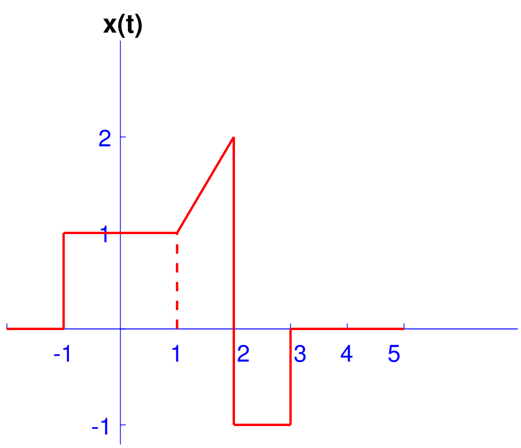


Figure 1.3: Problem 5.

Problem 6. Evaluate the following expressions using the unit impulse and unit step functions properties:

(a) $\int_0^3 \sin(t)\delta(3t-6)dt$

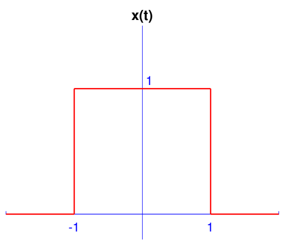
(b) $\int_0^4 2\cos(t-2)\delta^{'}(t-2)dt$

(c) $x(t) = t\delta^{'}(t) + \delta(t)$

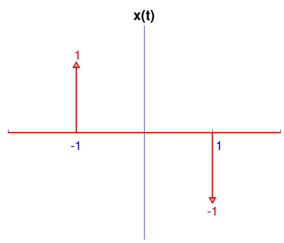
(d) $\int_{-\infty}^{\infty} \delta(4-t^2)dt$

(e) $x(t) = \int_{t-1}^{t-4} \tau^2\delta(t-\tau-2)d\tau$

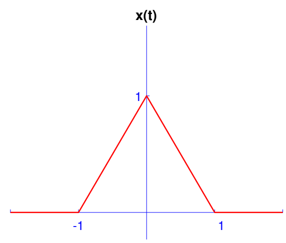
Problem 7. Determine and sketch signal $y(t) = \int_{-\infty}^{t} x(t)dt$ for the signals shown in the following figures.



(a)



(b)



(c)

Figure 1.7: Problem 7.

Problem 8. A continuous-time signal x(t) is shown in Fig. 1.8. Sketch and label each of the following signals.

(a) $y(t) = x(t)\,\mathrm{u}(t) + x(-t)\,\mathrm{u}(t)$

(b) $y(t) = x(2t-1)$

(c) $y(t) = x(-\frac{t}{2}+1)$

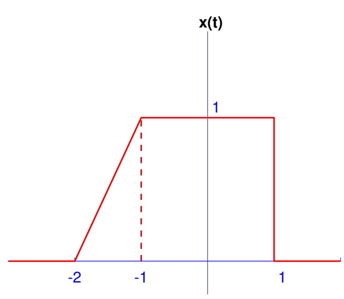


Figure 1.8: Problem 8.

Problem 9. Calculate the energy and power for the following continuous time signals and determine whether the signals are energy signals, power signals, or neither.

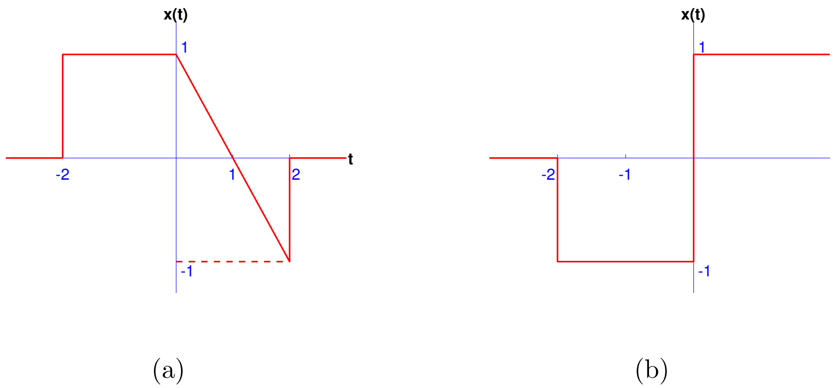


Figure 1.11: Problem 9.

Problem 10. Calculate energy and power for the following signal and determine whether the signal is energy signal, power signal, or neither.

$x(t) = \cos(t) + \cos(\alpha t)$, where $\alpha > 0$

Problem 11. Consider a continuous-time system with input $x(t)$ and output $y(t)$. The input-output relationship for this system is

$$y(t) = tx(3t) + 2.$$

Determine the output of the system when the input is $x(t-5)$.

Problem 12. Consider a continuous-time system with input $x(t)$ and output $y(t)$. The input-output relationship for this system is

$$y(t) = \begin{cases} x(t) - x(-t) & x(t) \geq x(-t) \\ x(-t) & x(t) < x(-t) \end{cases}.$$

For the continuous-time signal $x(t)$ shown in Fig. 1.12, determine the output of the system.

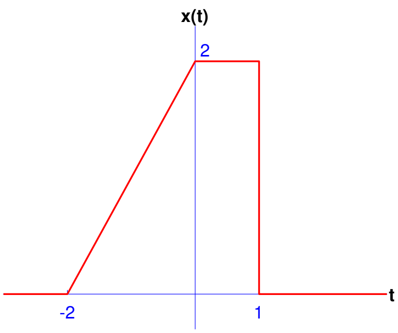


Figure 1.12: Problem 12.

Problem 13. Determine which of the following signals are periodic. If a signal is periodic, determine its fundamental period.

(a) $x[n] = \sin(\frac{7\pi}{3}n)$

(b) $x[n] = \cos(n\pi) + \cos(n)$

(c) $x[n] = e^{j(n+3)}$

(d) $x[n] = \cos(\frac{\pi}{4}n^2)$

Problem 14. Sketch the following signal and determine and sketch its even and odd components.

$$x[n] = 3\delta[n+4] - 2\delta[n+3] + u[n+2]$$

Problem 15. If the even component of the signal is shown in the following figure and for $n < 0$, $x[n] = 0$, find $x[n]$.

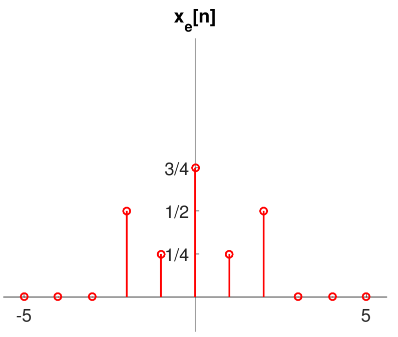
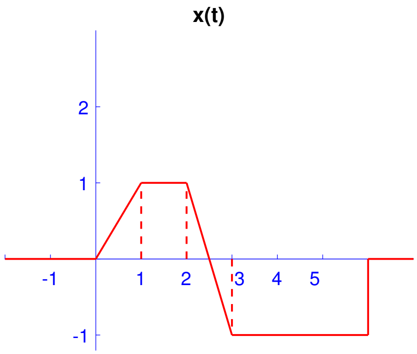
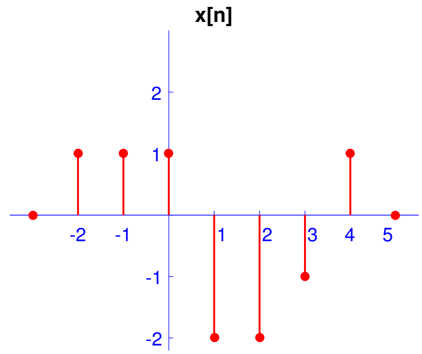


Figure 1.13: Problem 15, the even part of $x[n]$.

Problem 16. Write the following signals using basic signals.



(a)



(b)

Problem 17. Evaluate the following expression using the unit impulse and unit step functions properties.

(a) $x[n] = \sum_{k=-\infty}^{7} \delta[n+3-k]$ (b) $x[n] = \sum_{k=-\infty}^{n} 3^k \delta[4-k]$

Problem 18. A discrete-time signal $x[n]$ is shown in the following figure.
Sketch and label each of the following signals.

(a) $y[n] = x[-2n+1]$

(b) $y[n] = x_2[n+4]$

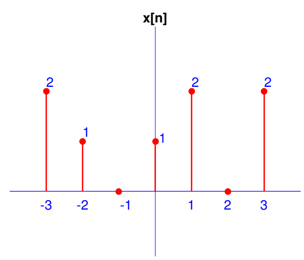


Figure 1.14: Problem 18.

Problem 19. Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = x[n^2+1] - x[-n^2+2n-1].$$

Determine the unit-step response of the system.

Problem 20. Calculate energy and power for the following signals and determine whether the signals are energy signals, power signals, or neither.

(a) $x[n] = \begin{cases} (\frac{1}{3} + j\frac{\sqrt{3}}{3})^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$

(b) $x[n] = (\frac{1}{3})^n \, \mathrm{u}[n]$

Chapter 2

Properties of Continuous-time and Discrete-time Signals

Problem 21. Determine whether the following continuous-time systems are linear.

(a) $y(t) = 3x(2t) + 3$

(b) $y(t) = \begin{cases} t^2 x(t+5) & t < 0 \\ 4sin(\sqrt{t})x(t^2) & t \geq 0 \end{cases}$

(c) $y(t) = \begin{cases} x(t) & x(t) < 1 \\ x(t) + 2 & x(t) \geq 1 \end{cases}$

(d) $y(t) = x(t^2) + x(t+5)$

(e) $y(t) = \text{Re}[x(t)]$

(f) $y(t) = \begin{cases} \frac{x^3(t-1)}{x^2(t-2)} & x(t-2) \neq 0 \\ 0 & x(t-2) = 0 \end{cases}$

(g) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(h) $y(t) = \sin(x(t))$

Problem 22. Consider the below continuous-time systems with the given input-output relationships. Determine whether these systems are time-invariant.

(a) $y(t) = 2x(t) - 1$

(b) $y(t) = x(t/4)$

(c) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(d) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

(e) $y(t) = x(1) + \int_{-\infty}^{t} x(\tau)d\tau$

Problem 23. Determine whether the continuous-time systems described by the following input-output equations are causal or noncausal.

(a) $y(t) = x(t/3)$

(b) $y(t) = \int_{-\infty}^{t/3} x(\tau)d\tau$

(c) $y(t) = x(t)x(-2)$

(d) $y(t) = \frac{dx(t)}{dt}$

(e) $y(t) = x(-|t|)$

Problem 24. Determine whether the following continuous-time systems are memoryless.

(a) $y(t) = x(-t)$

(b) $y(t) = e^{t+3}x(t)$

(c) $y(t) = \cos(x(t) + 1)$

(d) $y(t) = x(t) + x(2)$

Problem 25. Comment on the stability of the following continuous-time systems.

(a) $y(t) = 5x(t) + 2$

(b) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

(c) $y(t) = \sin(t)x(t)\delta(t-1)$

(d) $y(t) = \begin{cases} \frac{1}{x(t)} & |x(t)| > 0 \\ x(t+1) & |x(t)| \leq 0 \end{cases}$

(e) $y(t) = \begin{cases} \frac{1}{x(t)} & |x(t)| > 1 \\ x(t+1) & |x(t)| \leq 1 \end{cases}$

Problem 26. Determine if each of the following continuous-time systems is invertible.

(a) $y(t) = \frac{dx(t)}{dt}$

(b) $y(t) = \text{even}\{x(t)\}$

(c) $y(t) = x(t) - \cos(y(t-1))$

(d) $y(t) = \sin\big(x(t)\big)$

Problem 27. The outputs of a continuous-time linear system S to the inputs e^{j3t} and e^{-j3t} are $y_1(t)$ and $y_2(t)$, respectively.

(a) Obtain the output of the system S based on $y_1(t)$ and $y_2(t)$ to the input $x(t) = \cos(3t)$.

(b) Obtain the output of the system S based on $y_1(t)$ and $y_2(t)$ to the input $x(t) = \cos(3t-2)$.

Problem 28. Consider the continuous-time system shown in Fig. 2.1.

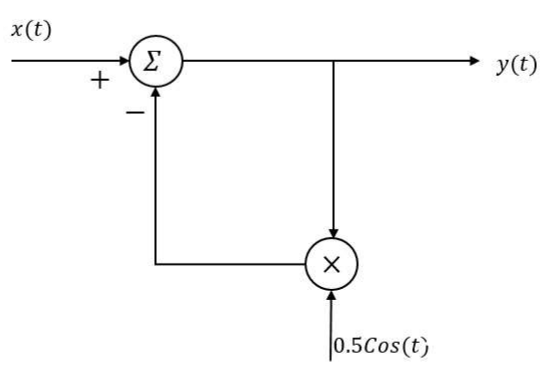


Figure 2.1: Problem 28.

(a) Find the relationship between the input and the output.

(b) Determine whether the system is (i) LTI, (ii) causal, (ii) memoryless, (iv) invertible, (v) stable.

Problem 29. Determine whether the continuous-time system shown in Fig. 2.2 is (a) linear, (b) time-invariant, (c) stable, (d) memoryless.

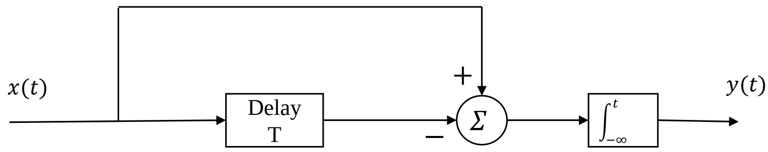


Figure 2.2: Problem 29.

Problem 30. Consider a continuous-time linear system with input $x(t)$ and output $y(t)$. Three inputs of this system and their corresponding outputs are shown in Fig. 2.3.

(a) Determine whether the system is (i) memoryless, (ii) causal, (iii) time-invariant.

(b) Determine and sketch carefully the response of the system to the input $x_4(t)$ depicted in Fig. 2.4.

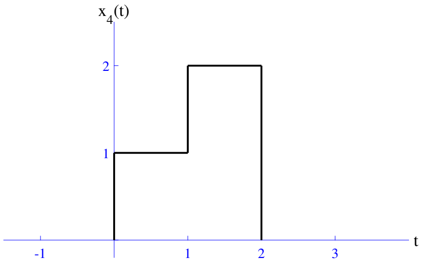


Figure 2.4: Problem 30 - Part (b), Signal $x_4(t)$.

Problem 31. Determine whether the following discrete-time system is linear.

$$y[n] = \begin{cases} x[n] & n > 10 \\ 5 & -10 \le n \le 10 \\ -x[n] & n < 10 \end{cases}$$

Problem 32. Consider the discrete-time systems with the following input-output relations. Determine whether these systems are time-invariant.

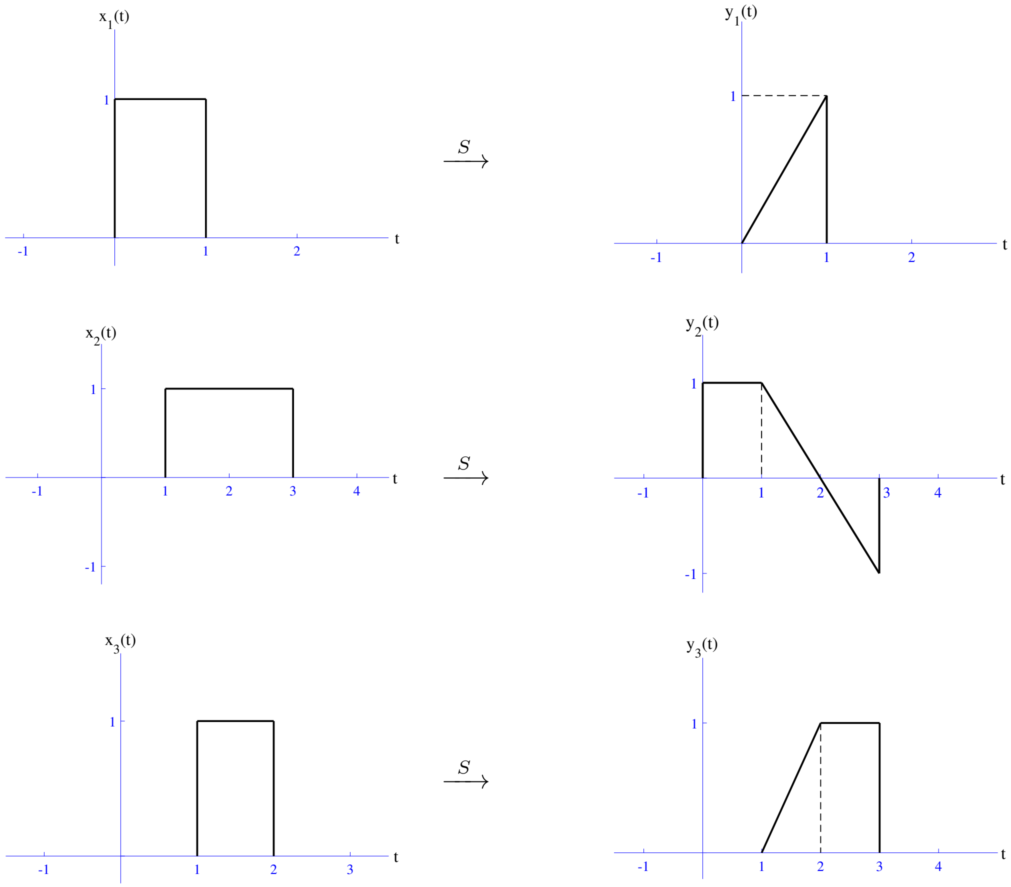


Figure 2.3: Problem 30 - Part (a).

(a) $y[n] = x[-n]$

(b) $y[n] = x^2[n] + n^2$

(c) $y[n] = \begin{cases} x^2[n] & x[n] \geq 0 \\ x^3[n+1] & x[n] < 0 \end{cases}$

(d) $y[n] = \begin{cases} x^2[n] & n \geq 0 \\ x^3[n+1] & n < 0 \end{cases}$

(e) $y[n] = \sum_{k=n}^{n+2} x[k]$

Problem 33. Determine if the discrete-time systems described by the following input-output equations are causal or noncausal.

(a) $y[n] = \begin{cases} x[n-2] + n + 7 & n \geq -1 \\ -x[n] - \cos(n) & n < -1 \end{cases}$

(b) $y[n] = \sum_{k=-\infty}^{n+4} x[k-1]2^{k+n+4}$

(c) $y[n] = \sum_{k=-\infty}^{n+5} x[k]\delta[k-n]k^2$

(d) $y[n] = \begin{cases} x^2[n] & x[-n+2] \geq 0 \\ x[n-1] & x[-n+2] < 0 \end{cases}$

Problem 34. Determine whether the following discrete-time systems are memoryless.

(a) $y[n] = y[n-3] + x[n]$

(b) $y[n] = x[n]\delta[n]$

(c) $y[n] = \sum_{k=-\infty}^{n+2} x[k-1]\delta[n-k]$

(d) $y[n] = \begin{cases} x[n] + \cos(n) & x[n-2] \geq 0 \\ x^3[n] + 3 & x[n-2] < 0 \end{cases}$

(e) $y[n] = \frac{1}{2N+1} \sum_{k=-N}^{N} x[n-k]$

Problem 35. Check the stability of following systems.

(a) $y[n] = x[2^{|n|}]$

(b) $y[n] = \sum_{k=n}^{n+5} x[5k+2]$

(c) $y[n] = \frac{\sqrt{|n|}x[n]}{5n-3}$

Problem 36. Determine if each of the following discrete-time systems is invertible.

(a) $y[n] = x[2n] + x[n]$

(b) $y[n] = \cos(\frac{\pi n}{7})x[n]$

(c) $y[n] = \begin{cases} x[n-1] & n \geq 1 \\ x^3[n] & n < 1 \end{cases}$

(d) $y[n] = \begin{cases} x[n/2] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

(e) $y[n] = x[2n]$

(f) $y[n] = x[n]x[n-3]$

Problem 37. Consider a discrete-time linear memoryless system with input / output relationships as follows:

$$x_1[n] = \delta[n] + \delta[n-1] + 2\delta[n-2] + 3\delta[n-3] + 4\delta[n-4], \tag{2.1}$$

$$y_1[n] = 2\delta[n-1] + 2\delta[n-2]. \tag{2.2}$$

(a) Find the output of this system to the input $x_2[n] = \delta[n-1] + \delta[n-2]$.

(b) What can we say about the output of this system to the input $x_3[n] = \delta[n-5]$?

Problem 38. Consider a discrete-time linear system S. The output of the system equals $y[n] = \cos(mn)$ when the input is $x[n] = u[n-m]$ for any integer m. Determine the output of the system when the input is as follows:

$$x[n] = \delta[n+1] + \delta[n] + \delta[n-1].$$

Problem 39. Consider a discrete-time linear system S. When the inputs to the system are $x_1[n]$, and $x_2[n]$, the outputs of the system are $y_1[n]$, and $y_2[n]$, as shown in Fig. 2.5, respectively. Determine whether the system is (a) memoryless, (b) time-invariant.

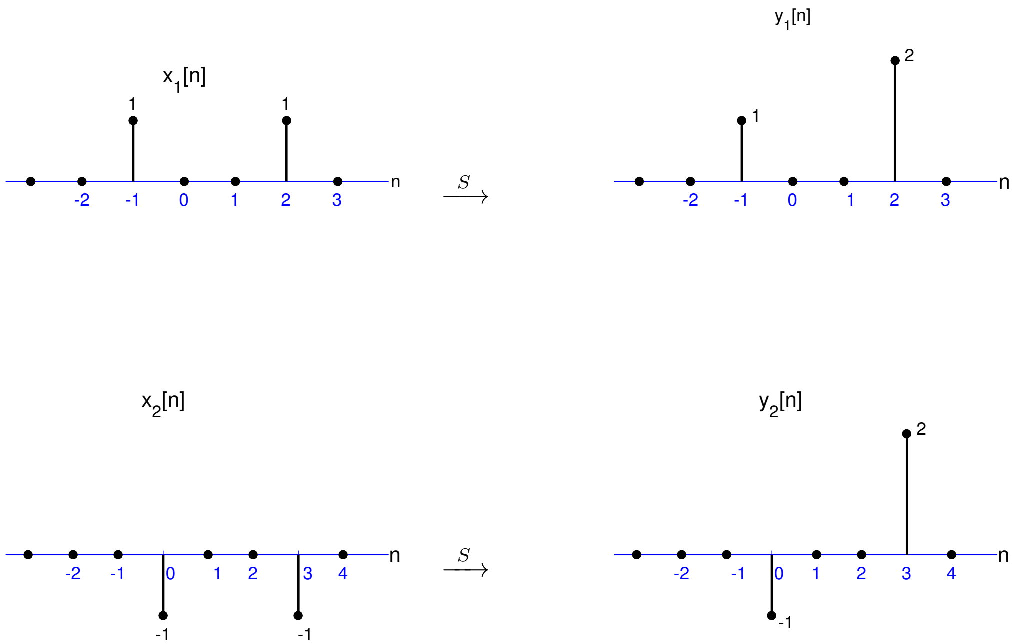


Figure 2.5: Problem 39.

Problem 40. Consider three systems with the following input-output relationships:

$$
\begin{aligned}
S_1: &\qquad z[n] = \begin{cases} x[n/2] & n = 2k \\ 0 & \text{otherwise} \end{cases}, \\
S_2: &\qquad w[n] = z[n] + z[n-1], \\
S_3: &\qquad y[n] = w[2n].
\end{aligned}
\tag{2.3}
$$

Suppose that these systems are concatenated in series. Is the overall system linear? Is it time-invariant?

Problem 41. Consider a linear system whose response to signal $x[n] = u[n-k]$ is the signal $y[n] = k^2\delta[n-k+1]$.

Is this system time-invariant?

Chapter 3

Liner Time-invariant Systems

Problem 42. Obtain and plot the convolution $y(t) = x(t) * h(t)$ for a continuous-time LTI system whose impulse response $h(t)$ and inputs $x_1(t)$ and $x_2(t)$ are given by

$$h(t) = \begin{cases} 1 & |t| \leq 1 \\ 0 & |t| > 0 \end{cases}.$$

(a) $x_1(t) = \begin{cases} 1 & 1 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases},$

(b) $x_2(t) = e^{-t}u(t),$

Problem 43. Compute the convolution of the following pairs of continuous-time signals.

(a) $x(t) = 2\sin(t)u(t), \quad h(t) = 2\cos(t)u(t)$

(b) $x(t) = tu(t), \quad h(t) = \sin(2t)u(t)$

(c) $x(t) = u(t-2), \quad h(t) = e^{-t}$

(d) $x(t) = \cos(t), \quad h(t) = e^{-|t|}$

Problem 44. Consider the following linear time-invariant system.

(a) The response of this system to signal $x_1(t)$ in Fig. 3.4 (a) is signal $y_1(t)$ illustrated in Fig. 3.4 (b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Fig. 3.4 (c).

(b) Find the impulse response $h(t)$ of this system and then work out Part (a) again.

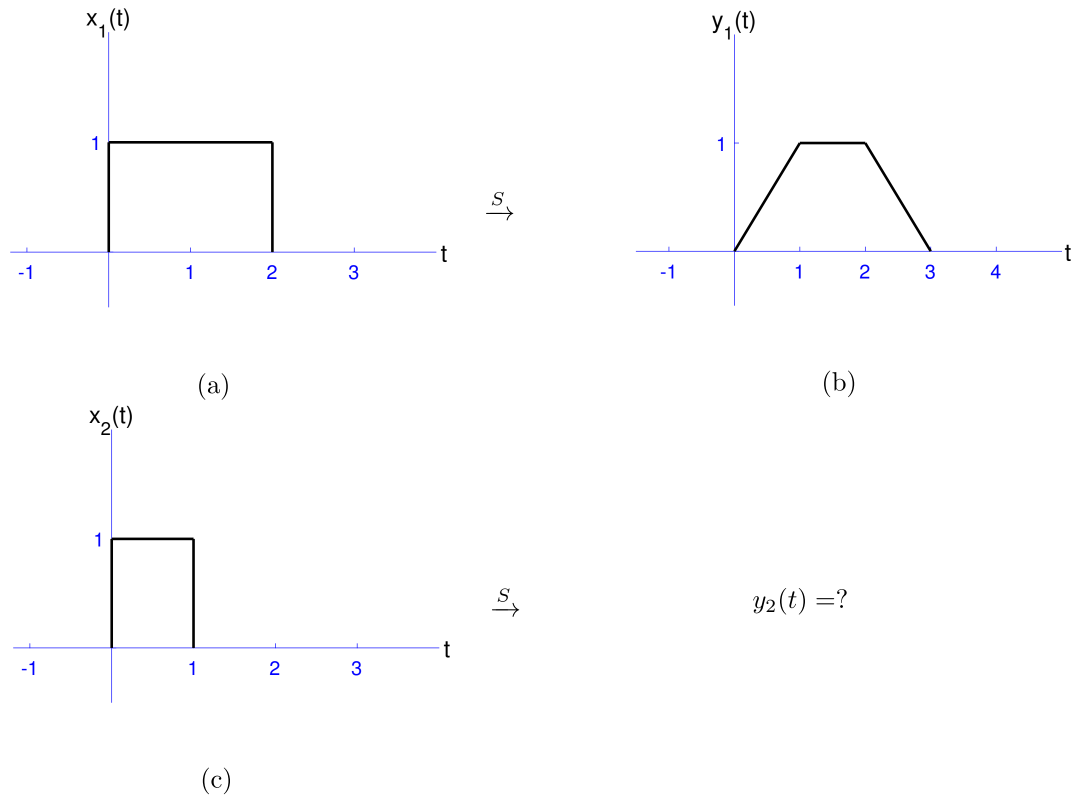


Figure 3.4: Problem 44.

Problem 45. Consider the following impulse responses of continuous-time LTI systems. Determine whether each system is (i) causal, (ii) memoryless, (iii) stable.

(a) $h(t) = t^2 e^{-t}\ u(t+2)$

(b) $h(t) = e^{t}\ u(-t-1)$

(c) $h(t) = e^{jat}\ u(2t+1)\ \ (a > 0)$

Problem 46. Consider the following integrals which represent systems with input $x(t)$ and output $y(t)$. Try to write each of the integrals as a convolution integral and then determine whether each system is LTI. Comment on system memory, causality, and stability.

(a) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau) d\tau$

(b) $y(t) = \int_{t+a}^{t+b} x(\tau) d\tau, \qquad b > a$

(c) $y(t) = \int_{0}^{+\infty} e^{-2\tau} x(\tau - t) d\tau$

(d) $y(t) = \int_{-\infty}^{-t} x(-\tau) d\tau$

Problem 47. Consider the system initially at rest condition, shown in Fig. 3.11.

(a) Find the input-output relationship for this system.

(b) Find the impulse response $h(t)$ of this system.

(c) ls this system memoryless?

(d) Is this system causal?

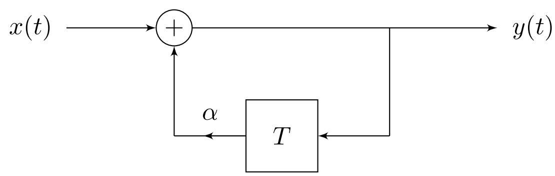


Figure 3.5: Problem 47.

Problem 48. For the following continuous-time LTI systems:
Plot the block diagram for the system described by the following differential equation:

$$\frac{dy}{dt} + ay(t) = \frac{dx}{dt} + bx(t). \tag{3.1}$$

Problem 49. Compute the convolution of the following pairs of discrete-time signals.

(a) $x[n] = (\frac{1}{6})^{n-5}u[n], \quad h[n] = (\frac{1}{5})^n u[n-2]$

(b) $x[n] = n(\frac{1}{2})^n \sin(n\pi - \frac{\pi}{2})u[n], \quad h[n] = u[n]$

Problem 50. A system is formed by connecting 3 systems as shown in Fig. 3.12. The impulse responses of the systems are given by

$$h_1[n] = \delta[n] + 3\delta[n-1], \quad h_2[n] = u[n] \quad \text{and} \quad h_3[n] = \delta[n-3].$$

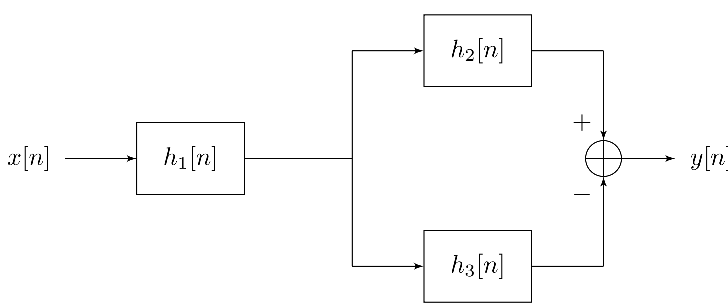


Figure 3.6: Problem 50.

Find the impulse response $h[n]$ of the overall system.

Problem 51. Consider the cascade of the following two discrete-time LTI systems S_1 and S_2, as depicted in Fig. 3.13. Let the step response of both systems be $s[n] = 3\delta[n-1] + \delta[n-2]$.

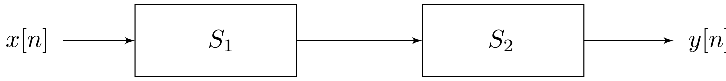


Figure 3.7: Problem 51.

(a) Find the impulse response of the overall system.

(b) Find the step response of the overall system.

Problem 52. Consider the following impulse responses of discrete-time LTI systems. Determine whether each system is (i) causal, (ii) memoryless and/or (iii) stable.

(a) $h[n] = (\frac{1}{3})^n u[n]$

(b) $h[n] = (1.001)^n u[n-3] - (\frac{1}{4})^n u[n]$

(c) $h[n] = n(\frac{1}{3})^n u[n-1]$

Problem 53. Consider a discrete-time LTI system S with the following impulse response $h[n]$:

$$h[n] = (\frac{1}{4})^n u[n], \tag{3.2}$$

Find the impulse response of the inverse system.

Hint: The system corresponds to the following difference equation:

$$h[n] - \frac{1}{4}h[n-1] = \delta[n]. \tag{3.3}$$

Problem 54. Consider a discrete-time system shown in Fig. 3.14.

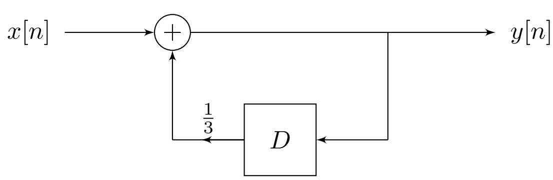


Figure 3.8: Problem 54, Block diagram of the system.

(a) Find the input-output relationship for this system.

(b) Find $y[n]$ with the initial condition $y[-1] = y_0$ and $x[n] = K\delta[n]$.

(c) For what conditions this system is LTI and causal.

Problem 55. Consider the following discrete-time LTI system. Plot the block diagram for this system.

$$y[n] - \frac{1}{3}y[n-1] = x[n] + x[n-1] \tag{3.4}$$

Problem 56. Write the difference equation for the block diagram shown in Fig. 3.15.

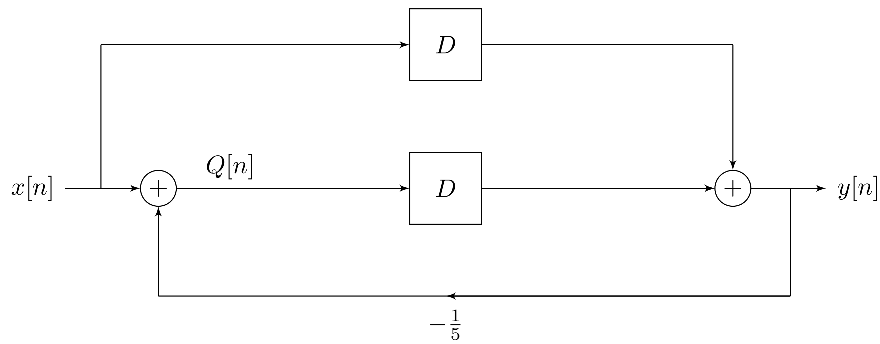


Figure 3.9: Problem 56, Block diagram of the system.

Problem 57. Consider the following discrete-time LTI systems.

(a) The step response $s_1[n]$ of the first system is given by

$$s_1[n] = \frac{3^n}{n+2}\cos(\frac{n\pi}{2})u[n]. \tag{3.5}$$

Determine the impulse response $h_1[n]$ of the system and the output of this system to the input $x_1[n]$ given by

$$x_1[n] = \delta[n] - \delta[n-1]. \tag{3.6}$$

(b) The step response $s_2[n]$ of the second system is given by

$$s_2[n] = (1 - (\frac{1}{2})^{n+1})u[n]. \tag{3.7}$$

Determine the impulse response $h_2[n]$ of the system and the output of this system to the input $x_2[n]$ given by

$$x_2[n] = (-1)^n u[n]. \tag{3.8}$$

Problem 58. The input $x[n]$ and the output $y[n]$ of a discrete-time LTI system are shown in Fig. 3.16 and Fig. 3.17, respectively.

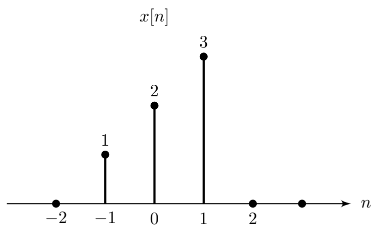


Figure 3.10: Problem 58, The input of the system.

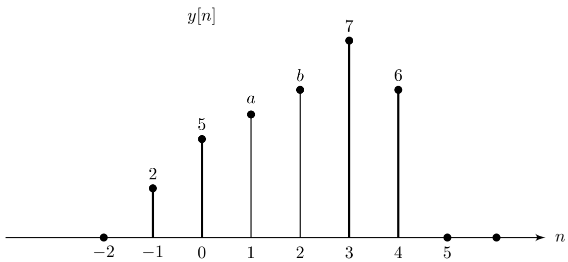


Figure 3.11: Problem 58, the output of the system.

(a) Find the impulse response for this system using the assumption that the length of the impulse response is 4.

(b) Find the values of the output $y[n]$ at $n = 1, 2$ (i.e. $y[1] = a$ and $y[2] = b$) using the impulse response $h[n]$ obtained in Part (a).

Problem 59. The system shown in Fig. 3.18 is formed by cascading two causal LTI systems. The input-output relationships for S_1 and S_2 are given by

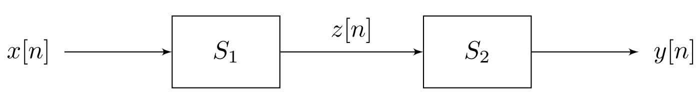


Figure 3.12: Problem 59, The cascade interconnection of two causal LTI systems.

$$S_1 : z[n] = \frac{1}{3}z[n-1] + x[n]$$
$$S_2 : y[n] = \alpha y[n-1] + \beta z[n]$$

and the input-output relationship of the overall system is

$$y[n] = -\frac{1}{6}y[n-2] + \frac{5}{6}y[n-1] + x[n]. \tag{3.9}$$

Find the values of α and β.

Problem 60. Consider a discrete-time LTI system where the input and output are related through the equation

$$y[n] = x[n] + \frac{1}{2}x[n-1]. \tag{3.10}$$

(a) Find the impulse response $h[n]$ of the system.

(b) Is the system causal?

(c) Find the output of this system to the input $x[n] = u[n+2]$.

(d) Consider the system below depicted in Fig. 3.19 in which the above system is a part of. Find the impulse response $h_{eq}[n]$ of this system assuming that

$$h_1[n] = h[n], \quad h_2[n] = \delta[n+1], \quad h_3[n] = -h[n-3].$$

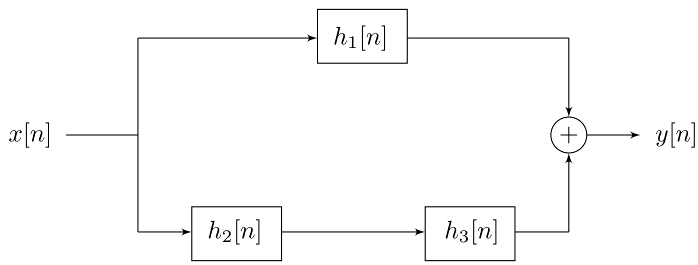


Figure 3.13: The LTI system for Problem 60 - Part (d).

(e) Consider the system of Part (d). Obtain the output of this system for the input $x[n] = u[n]$.

Problem 61. Consider a discrete-time LTI system whose input $x[n]$ and output $y[n]$ satisfy the second-order difference equation

$$y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n] - x[n-1]. \tag{3.11}$$

The system also satisfies the condition of initial rest.

(a) Find the impulse response $h[n]$ of the system.

(b) Determine the step response of the system.

Problem 62. Consider a discrete-time LTI system whose the impulse response is given by

$$h[n] = (\frac{1}{2})^n u[n]. \tag{3.12}$$

Determine the response of this system to the input

$$x[n] = \cos(n\pi)u[n].$$

Problem 63. Find the output $y[n]$ for the following difference equation.

$$y[n] + \frac{1}{3}y[n-1] = 4, \tag{3.13}$$

with the auxiliary condition $y[-1] = -3$.

Chapter 4

Fourier Series Representation of Continuous-time Signals

Problem 64. Determine and sketch the complex exponential Fourier series representation for each of the following signals

(a) $x(t) = \cos(t + \theta)$

(b) $x(t) = \sin(t) + \cos(t)$

(c) $x(t) = \sin^2(4t)$

(d) $x(t) = \cos(t). \sin(3t)$

(e) $x(t) = \cos^2(4t)$

Problem 65. Determine the complex exponential Fourier series representation for the following signal:

$$x(t) = \sum_{m=-\infty}^{\infty} (-1)^m [\delta(t - \frac{1}{3}m) + \delta(t + \frac{2}{3}m)]$$

Problem 66. Determine the complex exponential Fourier series representation of the following signals.

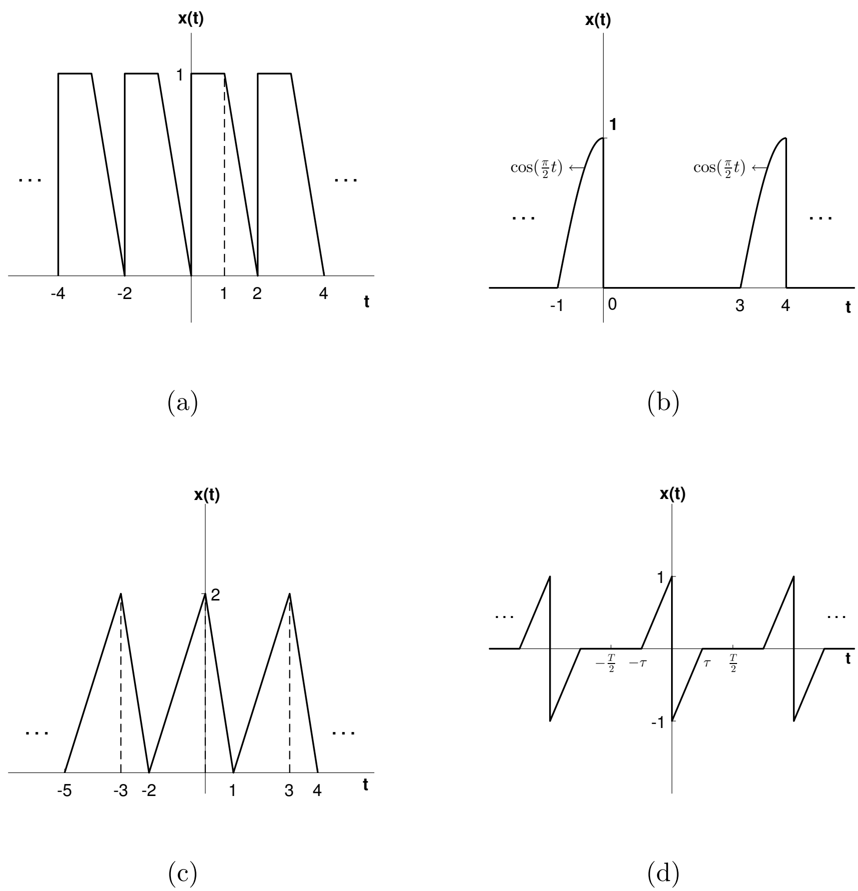


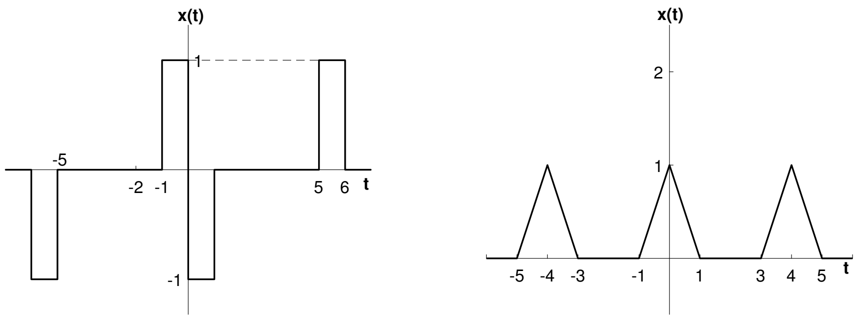
Figure 4.5: Problem 66.

Problem 67. A continuous-time periodic signal $x(t)$ is real valued and has fundamental period $T = 2$ and $x(t) = e^{-t}$ for $-1 < t < 1$.

(a) Determine the Fourier series coefficients of $x(t)$ through through direct evaluation.

(b) Is it possible to obtain the result in Part (a) without direct evaluatin?

Problem 68. Use the Fourier series properties to calculate the coefficients a_k for the following continuous-time periodic signals.



(a) (b)

Figure 4.8: Problem 68.

Problem 69. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 4$ ($w = \frac{\pi}{2}$). Determine $x(t)$ if the Fourier series coefficients of $x(t)$ are

$$a_k = \frac{\sin(\frac{k\pi}{4})}{k\pi}(2\cos(\frac{k\pi}{4}) + 2e^{-\frac{jk\pi}{4}}).$$

Problem 70. Let $x(t)$ be a periodic signal with period $T = 5$ where the Fourier series coefficients of the signal are given as follows:

$$a_{-2} = 3,\ a_{-1} = 1,\ a_0 = 3,\ a_1 = 1,\ a_2 = 3.$$

Evaluate the integrals in Parts (a) and (b).

(a) $A = \int_{-1}^{4} x(t)dt$ (b) $B = \int_0^5 x(t)e^{j\frac{4\pi}{5}t}dt$

Problem 71. In each of the following, we specify the Fourier series coefficients of a signal that is periodic. Determine $x(t)$ in each case.

(a) $a_1 = j,\ a_{-1} = -j,\ a_3 = 2,\ a_{-3} = 2,\ a_k = 0$, otherwise, $w_0 = 2\pi$

(b) $a_k = (-\frac{1}{5})^{|k|}$, $w_0 = 1$

(c) $a_k = \begin{cases} ke^{j\pi/2} & , |k| \leq 3 \\ 0 & , o.w. \end{cases}$, $w_0 = \frac{2\pi}{T}$

Problem 72. Suppose that $x(t)$ is a periodic signal with period T and Fourier series coefficients a_k.

(a) Let $T = 7$. Now let periodic signal $y(t)$ with period $T' = 7$ have Fourier series coefficients $b_k = a_{k-5}e^{jk\frac{4\pi}{7}}$. Write $y(t)$ based on $x(t)$.

(b) Let $T = 3$. Determine the periodic signal $y(t)$ with period $T' = 3$ and Fourier series coefficients $b_k = \text{Re}\{a_k\}$.

Problem 73. Let $x(t)$ be a periodic signal with period $T = 6$ and Fourier series coefficients a_k. Determine signal $y(t)$ with Fourier series coefficients $b_k = (-1)^{3k} a_k + (-1)^{3k} a_{-k}$.

Problem 74. Let $x(t) = |\cos(2t)|$ with Fourier series coefficients a_k. Compute the following expression:

$$A = \sum_{k=-\infty}^{\infty} |a_k|^2.$$

Problem 75. As shown in the Fig. 4.9, $x(t)$ is a periodic signal with period $T = 4$ and Fourier series coefficients a_k. Compute the following expression:

$$A = \sum_{k=-\infty}^{\infty} |ka_k|^2$$

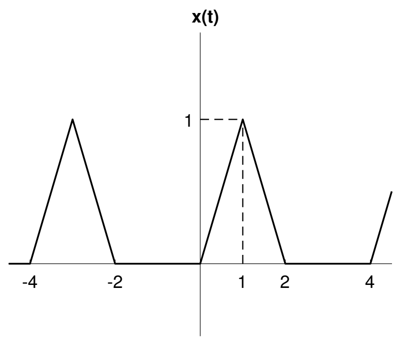


Figure 4.9: Problem 75, Periodic signal $x(t)$.

Problem 76. Consider signal $x(t)$ with a fundamental period of T_0. Determine the Fourier series coefficients for each of the following cases.

(a) $y(t) = x(1 + 3t)$

(b) $y(t) = x^*(5t - 3)$

(c) $y(t) = x(t) + x(-t)$, $x(t)$ is real

(d) $y(t) = x(t).\cos(\frac{2\pi t}{T_0})$

Problem 77. Consider signal $x(t)$ with a fundamental period of $T = 4$ and the Fourier series coefficients $F_k = \text{sinc}(k\frac{\pi}{2})$. Determine the Fourier series coefficients for each of the following cases.

(a) $y(t) = x(2t)$

(b) $y(t) = x^{'}(t)$

(c) $y(t) = x(t - \frac{1}{5})$

(d) $y(t) = \text{Re}\{x(t)\}$

(e) $y(t) = x(t).\cos(2\pi t)$

Problem 78. Suppose we are given the following facts about a signal $x(t)$:

(a) $x(t)$ is real and periodic signal with period $T = 4$, and it has Fourier series coefficients a_k.

(b) $a_k = 0$ for $k \geq 3$.

(c) the DC Fourier coefficient a_0 is zero.

(d) $x(t) = -x(t-2)$

(e) $\int_0^4 |x(t)|^2 dt = 2$

(f) a_1 is a negative real number.

Determine $x(t)$.

Problem 79. Suppose we are given a signal $x(t)$ with period $T = 2$, and it has Fourier series coefficients C_n where,

$$C_n = \begin{cases} 0, & n \text{ is even} \\ 1, & n \text{ is odd} \end{cases}.$$

The signal is entered into a filter with frequency response $H(jw)$, shown in the following figure. Find the output signal $y(t)$.

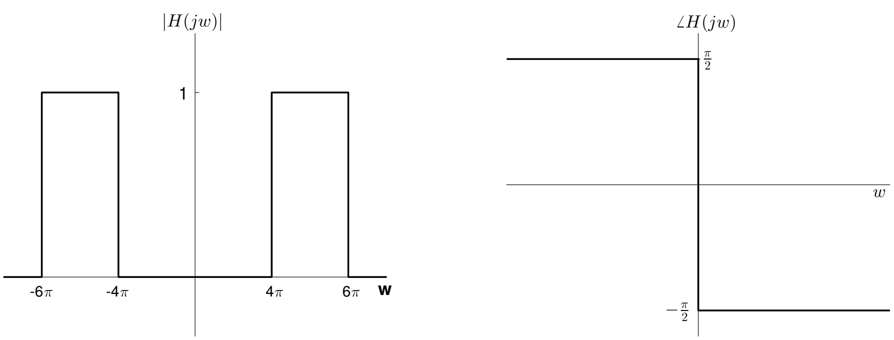


Figure 4.10: Problem 79, Magnitude and Phase of the considered filter $H(jw)$.

Problem 80. Consider signals $x(t)$ and $y(t) = x^*(t) + x(-\frac{t}{3})$ with the Fourier series coefficients a_k and b_k, respectively. Find the value of b_k based on a_k.

Problem 81. Suppose that periodic signal $x(t) = \cos(t)$ is the input signal to an LTI system with frequency response $H(jw)$. $H(jw)$ is demonstrated by its magnitude $|H(jw)|$ and phase $\angle H(jw)$ in figures below. Determine $y(t)$, the output of the system.

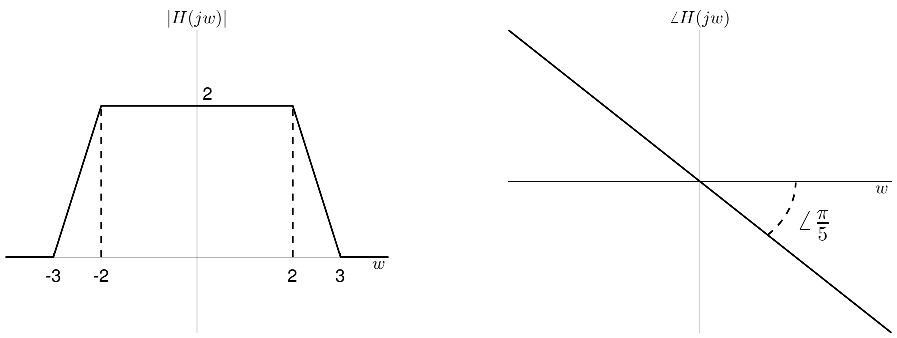


Figure 4.11: Problem 81, Magnitude and Phase of the frequency response $H(jw)$.

Chapter 5

Fourier Series Representation of Discrete-time Signals

Problem 82. Determine the fundamental period N_0 and frequency w_0 of the following signals. Then, find their Fourier series coefficients.

(a) $x[n] = A\cos(\frac{\pi n}{2} + \varphi)$

(b) $x[n] = \cos(\frac{\pi n}{2})\cos(\frac{3\pi n}{7})$

(c) $x[n] = 1 + 2\cos(\frac{\pi n}{3} + \frac{\pi}{4}) + \sin(\frac{\pi n}{6})$

(d) $x[n] = \sum_{m=-\infty}^{+\infty}(-1)^m\big[\delta[n-m] + \delta[n+3m]\big]$

(e) $x[n] = e^{j\frac{3\pi}{5}n} + \sum_{m=-\infty}^{+\infty}\delta[n-10m]$

Problem 83. $x[n]$ is a periodic signal with fundamental period N where

$$x[n] = 1 - \cos(\frac{n\pi}{4} + \frac{\pi}{4}) \qquad 0 \le n \le N-1.$$

Determine its Fourier series coefficients for $N = 4$.

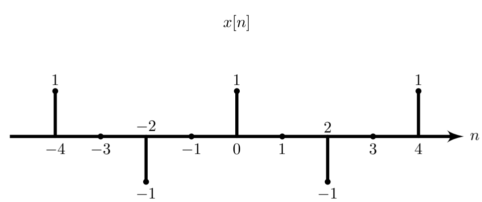
Problem 84. Determine the discrete Fourier series representation for each of the following signals.

(a) $x[n] = \cos(\frac{\pi}{3}n) + (-1)^n$

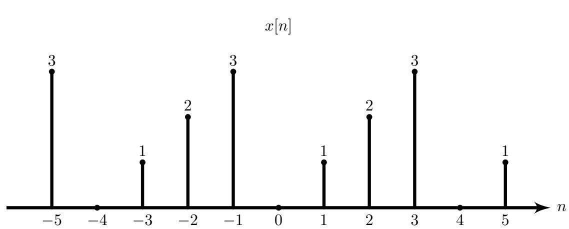
(b) $x[n] = (-1)^n + e^{jn(\pi/2)}$

(c) $x[n] = (j)^n$

Problem 85. Find the discrete Fourier series for each of the following periodic signals shown in Fig. 5.1.



(a)



(b)

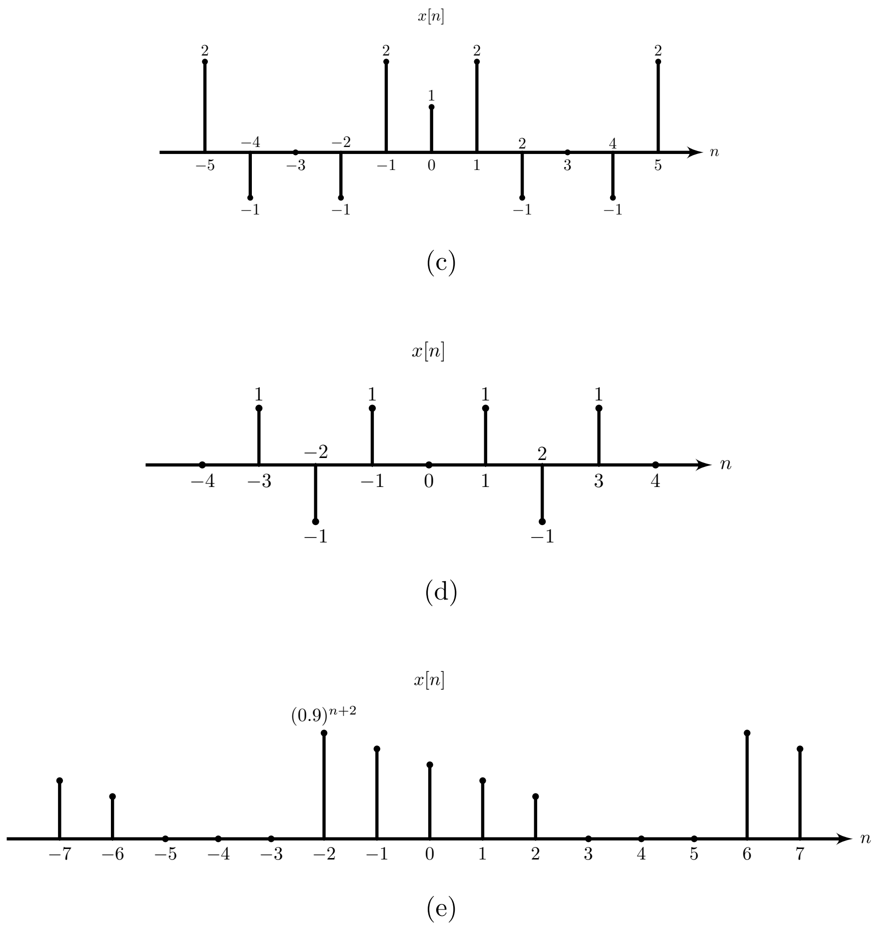


Figure 5.1: Problem 85

Problem 86. Consider a discrete-time LTI system. Find the corresponding system output $y[n]$, if

(a) The frequency response of this system is

$$H(e^{jw}) = \frac{\cos(\frac{3}{2}w)}{\cos(\frac{w}{2})} + 1$$

and the input is

$$x[n] = \sum_{k=-2}^{2} |k| e^{jk(2\pi/3)n}.$$

(b) The frequency response is

$$H(e^{jw}) = 4\cos(2w)$$

and the input to this system is

$$x[n] = \begin{cases} 0 & n \text{ even} \\ 1 & n \text{ odd} \end{cases}.$$

Problem 87. As shown in the following figure, $x[n]$ is a periodic signal with period $N = 5$ and the Fourier series coefficients a_k.

(a) Find the value of

$$\sum_{k=0}^{4} |a_k|^2.$$

(b) Find the value of

$$\sum_{k=0}^{4} (Re\{a_k\})^2.$$

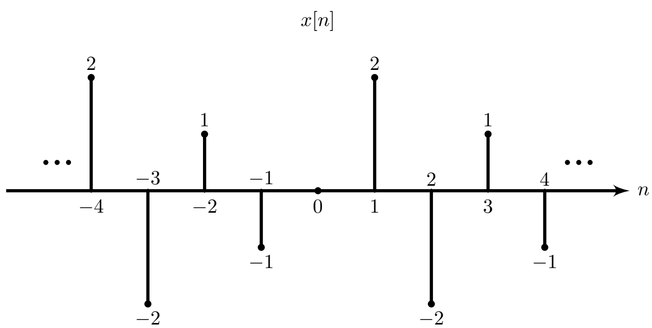


Figure 5.2: Problem 87, Discrete-time periodic signal $x[n]$.

Problem 88. Consider the discrete-time signal $x[n]$ with period N and the Fourier series coefficients a_k. Determine the Fourier series coefficients for each of the following cases based on a_k.

(a) $y[n] = x[n]\cos(\frac{2\pi}{N}n)$

(b) $y[n] = (-1)^n x[n], \quad N = \text{even}$

(c) $y[n] = x_2[n-1]$

(d) $y[n] = x^*[-n+1], \quad N = 8$

Problem 89. Let $x[n]$ be a discrete-time periodic signal with period N and Fourier coefficients a_k.

(a) If the period is $N = 3$ and the Fourier series coefficients are $a_k = j^k$ where $j = \sqrt{-1}$, determine the power of the signal $y[n] = 2x[n] + 2$.

(b) Now assume that $x[n]$ is another signal with period $N = 4$ which is real and has the average value of zero. Given that

$$a_1 = -1 + j, \qquad a_2 = -2,$$

determine the power of $x[n]$.

Problem 90. Suppose we are given the following facts about a signal $x[n]$:

1. $x[n]$ is a real and even signal.

2. $x[n]$ is periodic with period $N = 6$ and has Fourier coefficients a_k.

3. $a_7 = 2$.

4. $\frac{1}{6}\sum_{n=0}^{5}|x[n]|^2 = 8$.

Using the above information, find $x[n]$.

Problem 91. Suppose a discrete-time periodic signal $x[n]$ has fundamental period N and Fourier coefficients a_k.

(a) Let us define $y[n]$ as

$$y[n] = 2x^*[n] + 2x_2[n]$$

with the Fourier coefficients b_k, where $x_2[n]$ is the time-scaled version of $x[n]$. If the period is $N = 3$, obtain the Fourier series coefficients of $y[n]$ based on a_k's.

(b) Signal $y[n]$ is defined as

$$y[n] = x[n]\cos(\frac{n\pi}{2})$$

If the period of $x[n]$ is $N = 4$, obtain the Fourier series coefficients of $y[n]$ based on a_k's.

Problem 92. In each of the following, we specify the Fourier series coefficients of a periodic discrete-time signal. Determine signal $x[n]$ in each case.

(a) $a_k = \cos(\frac{4\pi}{23}k)$

(b) $a_k = \sum_{m=-\infty}^{+\infty}(-1)^m\big(\delta[k-2m] + \delta[k+5m]\big)$

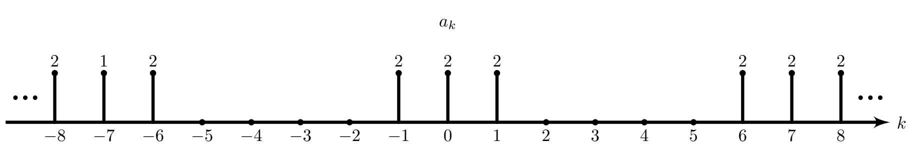


Figure 5.3: Problem 92 - Part (c)

Problem 93. Consider signal $x[n]$ depicted in Fig. 5.4. This signal is periodic with period $N = 4$ and has the Fourier series coefficients a_k. Determine signal $z[n]$ corresponding to each of the following Fourier series coefficients.

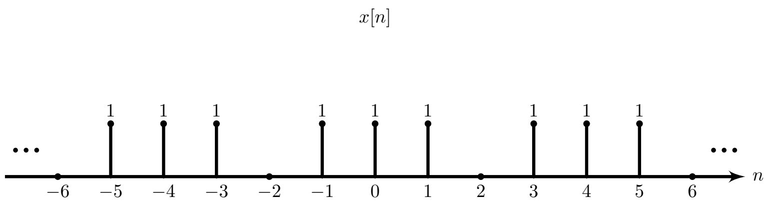


Figure 5.4: Problem 93, Periodic Signal $x[n]$.

(a) $(-1)^k a_k$

(b) a_{k-2}

Problem 94. Let $x[n]$ be a real and even periodic signal with fundamental period $N = 10$ ($w_0 = \frac{\pi}{5}$) and power $P = 3/2$. We also know that the maximum value of $x[n]$ is equal to 2. This signal enters a filter with frequency response $H(jw)$, shown in Fig. 5.5. The output signal has the average power equal to the power of $x[n]$. Find signal $x[n]$.

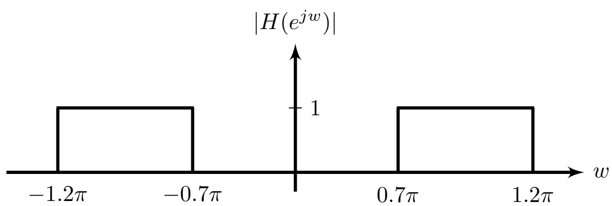


Figure 5.5: Problem 94, Frequency response of the ideal band-pass filter.

Problem 95. A discrete-time periodic real signal $x[n]$ has period $N = 5$ and Fourier coefficients a_k. Let

$$a_0 = 2, \quad a_2 = e^{j(\pi/5)}, \quad a_4 = e^{j(\pi/4)}.$$

(a) Determine the values of a_1, a_{-1} and a_{-2}.

(b) Determine signal $x[n]$ using the result of Part (a).

Problem 96. As shown in Fig. 5.6, $x[n]$ is a periodic signal with the Fourier series coefficients a_k. Find the value of

$$I = \sum_{k=-6}^{5} a_k.$$

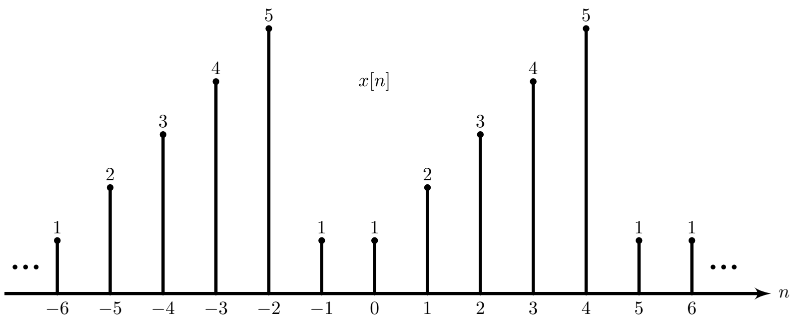


Figure 5.6: Problem 96, Periodic signal $x[n]$.

Problem 97. Consider a discrete-time LTI system whose frequency response is

$$H(e^{jw}) = \cos(4w).$$

If the input to this system is the following periodic signal

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k],$$

find the corresponding system output $y[n]$.

Problem 98. Consider a discrete-time periodic signal $x[n]$ with period N and Fourier series coefficients a_k.

(a) If $x[n]$ is as shown in Fig. 5.7 with period $N = 4$, determine signal $y[n]$ at time $n = 2$ with Fourier series coefficients $b_k = (a_k)^2$.

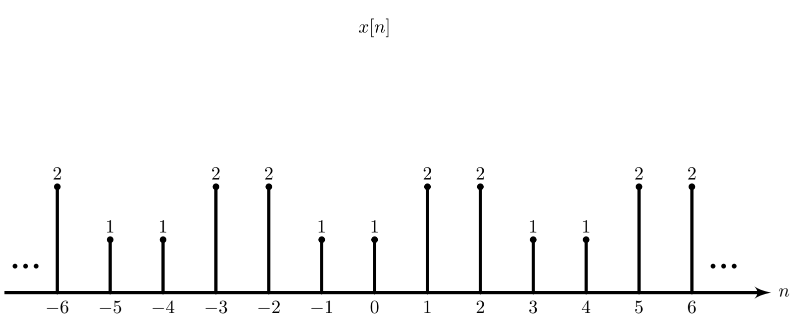


Figure 5.7: Problem 98 - Part (a), Periodic signal $x[n]$.

(b) If a_k as shown in Fig. 5.8 with period $N = 8$ and we let $y[n] = (x[n])^2$ with Fourier series coefficients b_k, determine the value of b_0.

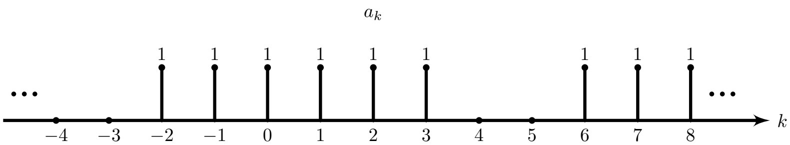


Figure 5.8: Problem 98 - Part (b), Fourier series coefficients a_k.

Chapter 6

The Continuous-time Fourier Transform

Problem 99. Use the Fourier transform definition to obtain the Fourier transform of the following signals:

(a) $x(t) = \delta(t - t_0)$

(b) $x(t) = \prod(\frac{t-2}{2})$

(c) $x(t) = e^t[\mathrm{u}(t) - \mathrm{u}(t-1)]$

(d) $x(t) = t\,\mathrm{u}(t)$

(e) $x(t) = \mathrm{u}(t+2) - 2\,\mathrm{u}(t) + \mathrm{u}(t-2)$

(f) $x(t) = \sum_{m=0}^{\infty} a^m \delta(t-m),\ m < 1$

(g) $x(t) = e^t\,\mathrm{u}(-t) + e^{-t}\,\mathrm{u}(t)$

(h) $x(t) = t^2 \prod(\frac{t}{4})$

Problem 100. For each of the following signals, use Fourier transform properties to determine the corresponding Fourier transforms.

(a) $x(t) = \cos(w_0 t)\,\mathrm{u}(t)$

(b) $x(t) = \frac{1}{\pi t}\cos(4\pi t).\sin(2\pi t)$

(c) $x(t) = \sin(|t|)$

(d) $x(t) = te^{-3|t-2|}$

(e) $x(t) = t^2 e^t\,\mathrm{u}(-t)$

(f) $x(t) = \begin{cases} 1 - t^2 & |t| < 1, \\ 0 & |t| > 1 \end{cases}$

(g) $x(t) = \left[\frac{2\sin(3t)}{\pi t}\right].\left[\frac{\sin(t)}{t}\right]$

Problem 101. Use the definition to obtain the Fourier transform of each of the following signals.

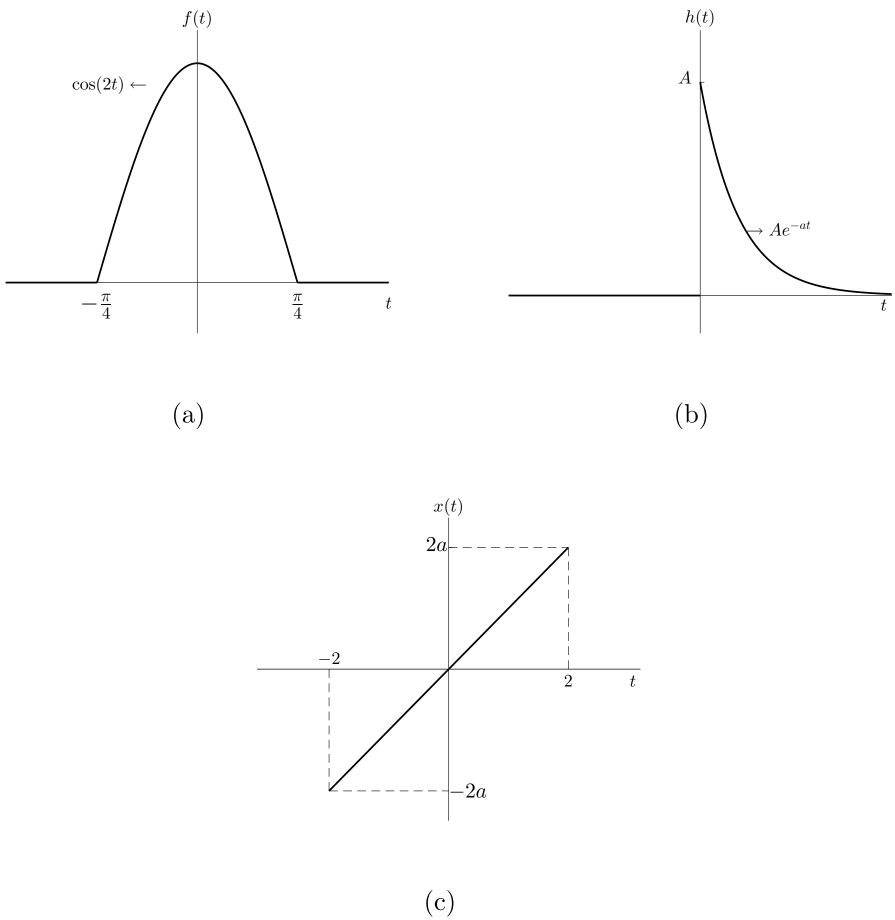


Figure 6.4: Problem 101.

Problem 102. For each of the following signals, use Fourier transform properties to determine Fourier transforms.

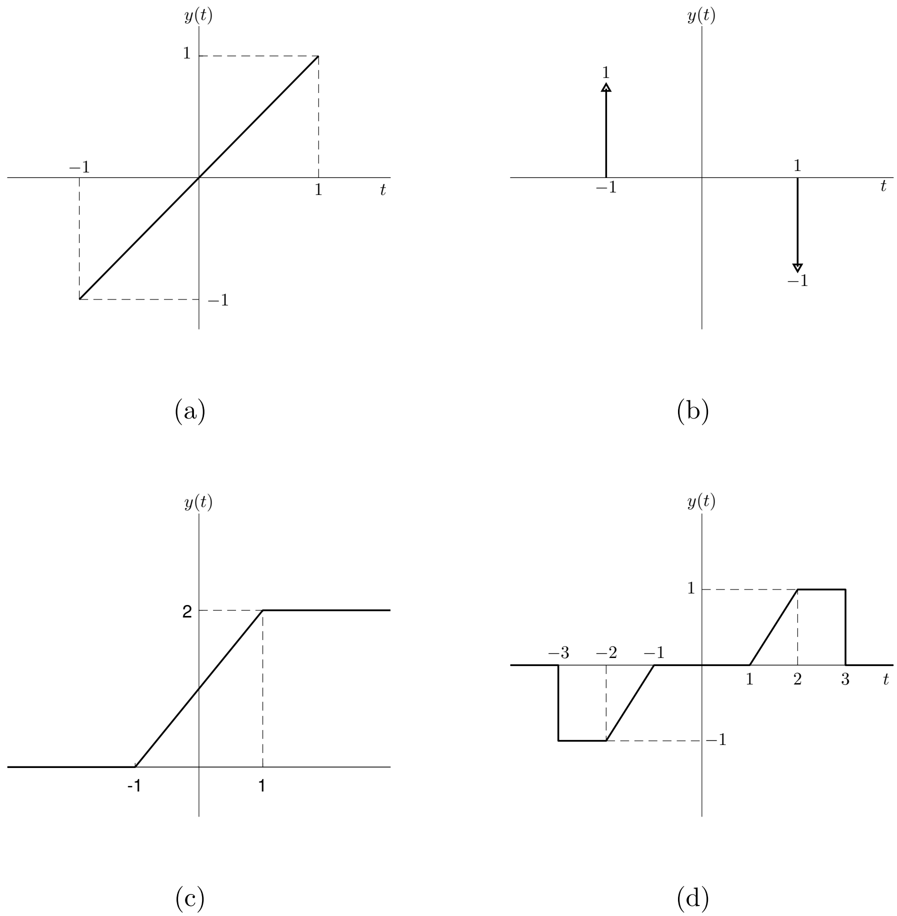
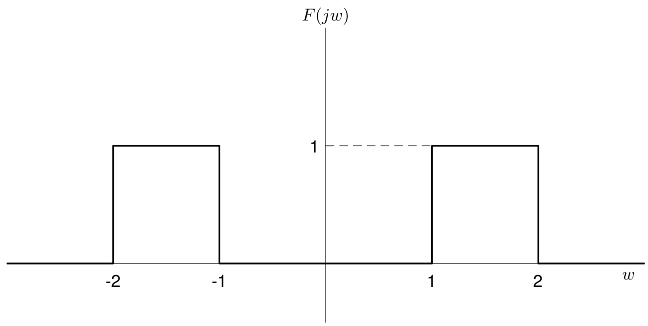


Figure 6.9: Problem 102.

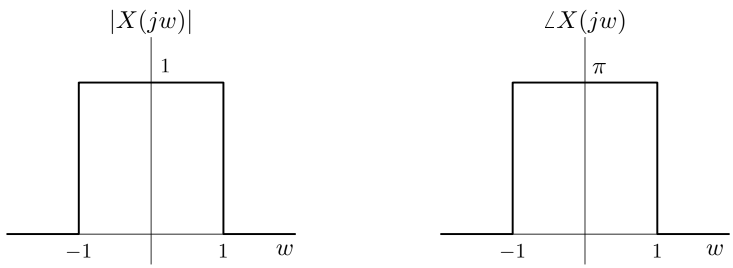
Problem 103. Use the definition to determine the inverse Fourier transforms of the following items.

(a) $F(jw) = e^{-|w|}$

(b) $F(jw) = \begin{cases} j\sin(w) & |w| < \pi \\ 0 & o.w \end{cases}$



(c)



(d)

Figure 6.12: Problem 103 - Parts (c) and (d).

Problem 104. Consider a real signal $x(t)$ where

$$X(jw) = \frac{1}{2 + j\frac{w}{3}}.$$

Determine the Fourier transform of each of the following signals.

(a) $x_a(t) = \frac{dx(t)}{dt}$

(b) $x_b(t) = x(-1-t)$

(c) $x_c(t) = e^{-jt}x(t-2)$

(d) $x_d(t) = x(t)\cos(2\pi t)$

Problem 105. For each of the following signals, use Fourier transform properties to determine the inverse Fourier transform.

(a) $X(jw) = \frac{e^{2jw}}{(1+jw)^2}$

(b) $X(jw) = e^{-w}\,\mathrm{u}(w)$

(c) $X(jw) = \frac{d}{dw}\{\frac{\sin(2w)-jcos(2w)}{1+j(\frac{w}{3})}\}$

(d) $X(jw) = 2\frac{w\cos(w)-\sin(w)}{w^2}$

(e) $X(jw) = \frac{4\sin^2(w)}{w^2}e^{-j2w}$

Problem 106. For each of the following signals, use Fourier transform properties to determine whether the corresponding Fourier transform is complex, real and even, imaginary and odd, conjugate symmetric, or neither.

(a) $x(t) = e^{-|t-2|}$

(b) $y(t) = \prod\left(\frac{t}{2}\right)$

(c) $z(t) = \sin(2\pi t)$

(d) $w(t) = \cos(2\pi t - \pi/4)$

(e) $b(t) = e^{j2|t|}$

Problem 107. $x(t)$ is as depicted in the Fig. 6.13.

(a) Evaluate $a = \int_{-\infty}^{\infty} X(jw)dw$

(b) Evaluate $b = \int_{-\infty}^{\infty} |X(jw)|^2 dw$

(c) Evaluate $c = \int_{-\infty}^{\infty} X(jw)e^{4jw}dw$

(d) Evaluate $d = X(j0)$

(e) Find $\angle X(jw)$

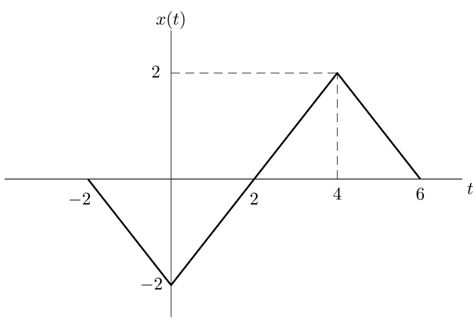


Figure 6.13: Problem 107, Signal $x(t)$.

Problem 108. For the following frequency response, determine a differential equation relating the input $x(t)$ and output $y(t)$.

$H(jw) = \frac{2+3jw-4(jw)^2}{1+jw}$

Problem 109. Evaluate the following integrals using the Fourier transform properties:

(a) $\int_{-\infty}^{\infty} \frac{\sin(w)\cos(wt)}{w} dw$

(b) $\int_{0}^{\infty} \frac{\sin(w)}{w} dw$

(c) $\int_{0}^{\infty} \frac{2}{(x^2+4)^2} dx$

(d) $\int_{-\infty}^{\infty} \frac{4x^2}{(x^2+4)^2} dx$

Problem 110. Consider an LTI system with impulse response

$$h(t) = 2\frac{\sin(\frac{\pi t}{2})}{\pi t}\cos(5\pi t).$$

Determine the output of the system for each of the following inputs:

(a) $x(t) = \cos(2\pi t) + \sin(5\pi t)$

(b) $x(t) = \sum_{m=-\infty}^{\infty} (-1)^m \delta(t-m)$

(c)

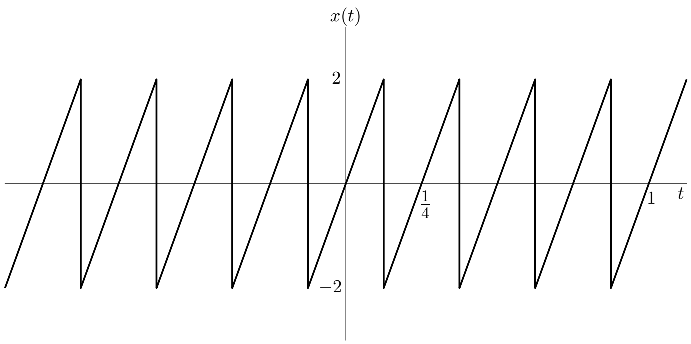


Figure 6.14: Problem 110 - Part (c).

Problem 111. The output $y(t)$ of a system is related to the input $x(t)$ by the equation

$$y(t) = \int_{t}^{t+a} x(\lambda)d\lambda - \int_{t-a}^{t} x(\lambda)d\lambda, \tag{6.1}$$

where a is a positive constant.

(a) Find the frequency response of this system.

(b) Find the response $y(t)$ to the input

$$x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \delta(t - \frac{ak}{2}).$$

Problem 112. For each of the following signals express the Fourier transforms of the signals in terms of the Fourier transform of $x(t)$.

(a) $y(t) = x(2t-4)$

(b) $y(t) = x^*(t-1)$

(c) $y(t) = x^*(1-\frac{t}{3})$

(d) $y(t) = x(2-t) + x(-2-t)$

(e) $y(t) = \int_{-\infty}^{\infty} x(\tau)x(\tau - t)dt$, x(t) is real

Problem 113. Consider an LTI system whose response to the input

$$x_1(t) = je^{j100\pi t}$$

is

$$y_1(t) = 2e^{j(100\pi t + \frac{\pi}{4})}.$$

The frequency response of system $|H(jw)|$ is zero except for $0 \le w \le 200\pi$ where it has the form $|k|e^{j\theta}$ where k and θ are constant. Determine the output of the system to each of the following inputs:

(a) $x_2(t) = (1+j)e^{j50\pi t}$

(b) $x_3(t) = \sqrt{2}e^{j300\pi t}$

Problem 114. A signal $x(t)$ has a Fourier transform $X(jw)$ as illustrated in Fig. 6.15. Evaluate the following without obtaining $x(t)$ directly.

(a) Evaluate $x(0)$

(b) Evaluate $\int_{-\infty}^{\infty} |x(t)|^2 dt$

(c) Evaluate $\int_{-\infty}^{\infty} x(t)\ dt$

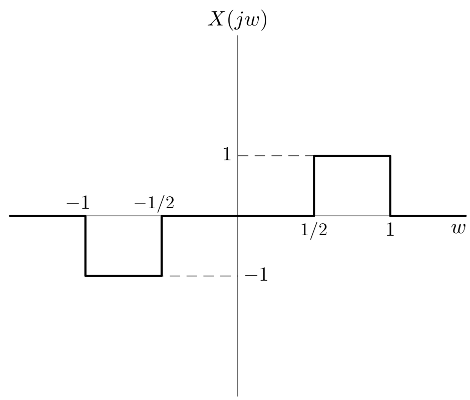


Figure 6.15: The Fourier transforms considered in Problem 114.

Problem 115. Evaluate the following integrals using Fourier transform properties.

(a) $A = \int_{-\infty}^{\infty} \frac{(1-\cos^2(t))^2}{\pi^4 t^4} dt$

(b) $B = \int_{-\infty}^{\infty} \frac{\text{sinc}(f)}{1+j2\pi f} df$

Problem 116. Consider the block diagram of a continues-time system in Fig. 6.16.

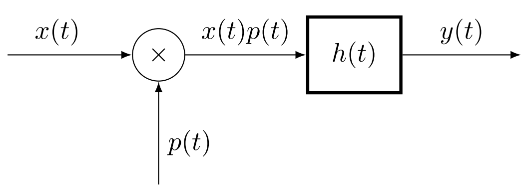


Figure 6.16: Block diagram of a continues-time system in Problem 116.

In this system assume that

$$x(t) = 2\frac{\sin(6\pi t)}{\pi t}e^{j2\pi t}$$

and

$$p(t) = \cos(4\pi t).$$

Also assume $h(t)$ has a Fourier transform $H(jw)$ whose magnitude and phase are as illustrated in Fig. 6.17.

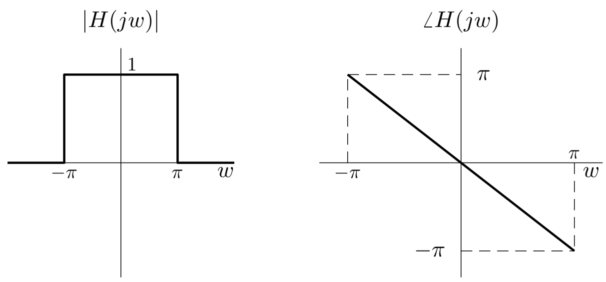


Figure 6.17: Magnitude and phase of the Fourier transform $H(jw)$ for the filter $h(t)$, Problem 116.

(a) Find $Y(jw)$.

(b) Find $y(t)$.

Problem 117. Determine if each of the following systems is time-invariant. In each case, y(t) denotes the system output and x(t) is the system input. Also $Y(jw)$ and $X(jw)$ denote the corresponding Fourier transforms, respectively.

(a) $y(t) = x(0) + X(t-5)$

(b) $y(t) = X(0) + x(t-5)$

(c) $Y(jw) = X(jw) + X(0)$

Problem 118. Consider a continuous-time system whose input $x(t)$ and output $y(t)$ are related through the following relationship:

$$y(t) = \int_{-\infty}^{\infty} \frac{x(\alpha)}{\pi(t-\alpha)}\, d\alpha.$$

If $y(t)$ is the response of this system to $x(t) = e^{-t}u(t)$, find $Y(jw)$.

Problem 119. A continuous-time signal

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - \frac{k\pi}{2})$$

is the input to a filter with frequency response $H(jw)$ whose magnitude and phase are as illustrated in Fig. 6.18.

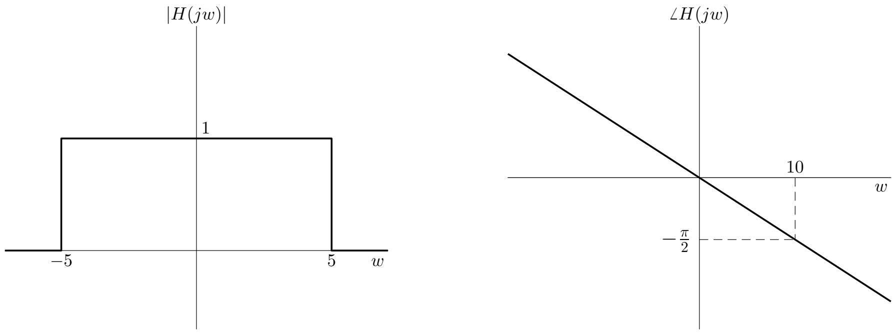


Figure 6.18: Problem 119, magnitude and phase of the Fourier transform for the filter $h(t)$.

Determine the filtered output signal $y(t)$.

Problem 120. Consider an LTI system whose response to the input

$$x(t) = \cos(w_0 t)$$

is

$$y(t) = 4e^{-2|w_0|}\cos(w_0 t + \frac{\pi}{2}),$$

where $w_0 \in \mathcal{R}$. Determine the system's impulse response.

Problem 121. A continuous-time LTI system has a frequency response $H(jw)$ whose magnitude and phase are as illustrated in Figure 6.19. Find the step response $s(t)$ of the system.

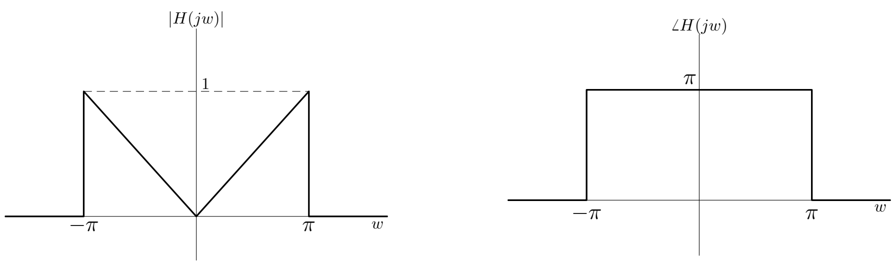


Figure 6.19: Problem 121, Frequency response $H(jw)$.

Chapter 7

The Discrete-time Fourier Transform

Problem 122. Use the discrete-time Fourier transform analysis definition to compute the Fourier transforms of each of the following signals:

(a) $x[n] = (\frac{3}{5})^n u[n-3]$

(b) $x[n] = a^{|n|}, \quad |a| < 1$

(c) $x[n] = \delta[4-2n]$

(d) $x[n] = (\frac{1}{3})^{-n} u[-n-1]$

(e) $x[n] = \sin(\frac{5\pi}{4}n) + \cos(\frac{3\pi}{4}n)$

Problem 123. Use the inverse Fourier transform formulation to compute the inverse Fourier transforms of the following signals.

(a) $X(e^{jw}) = \cos^2(w)$

(b) $X(e^{jw}) = \cos(w) + j\sin(w)$

(c) As in Fig. 7.1.

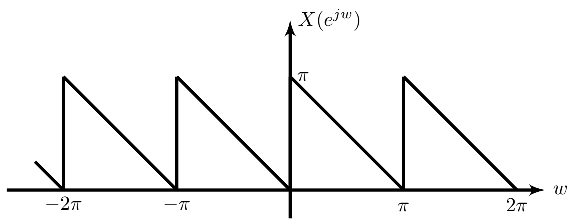


Figure 7.1: Problem 123 - Part (c).

(d) As in Fig. 7.2 where $\angle X(e^{jw}) = 0 \, \forall w$.

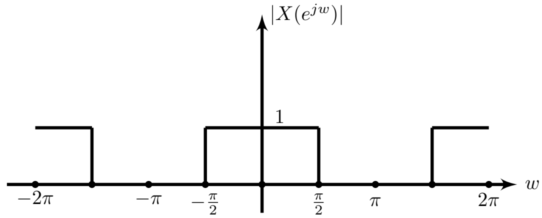


Figure 7.2: Problem 123 - Part (d).

Problem 124. For each of the following signals, use Fourier transform properties to determine the corresponding Fourier transform.

(a) $x[n] = (\frac{1}{3})^n u[n+3]$

(b) $x[n] = \cos(\frac{\pi}{2}n)(\frac{1}{3})^n u[n-1]$

(c) $x[n] = \dfrac{\sin(\frac{\pi}{2}n)}{n\pi} * \dfrac{\sin(\frac{\pi}{2}(n-3))}{(n-3)\pi}$

(d) $x[n] = na^n u[n], \qquad |a| < 1$

Problem 125. For each of the following Fourier transforms, use Fourier transform properties to determine the inverse Fourier transform.

(a) $X(e^{jw}) = j\sin(2w)$

(b) $X(e^{jw}) = \dfrac{1}{1 - \frac{1}{2}e^{-j5w}}$

(c) $X(e^{jw}) = \cos(2w)\left[\dfrac{\sin(\frac{5w}{2})}{\sin(\frac{w}{2})}\right]$

(d) $X(e^{jw}) = e^{-j(2w+\pi/2)}\frac{d}{dw}\left[\dfrac{2}{1 - \frac{1}{3}e^{-j(w-\pi/3)}} + \dfrac{2}{1 - \frac{1}{3}e^{-j(w+\pi/3)}}\right]$

(e) $X(e^{jw}) = \dfrac{10}{-e^{-j2w} + 3e^{-jw} + 10}$

Problem 126. Consider the discrete-time signal $x[n]$ with Fourier transform $X(e^{jw})$. Determine the Fourier transforms for each of the following cases in terms of $X(e^{jw})$.

(a) $y[n] = x^*[-n]$

(b) $y[n] = x[n] * x^*[-n]$

(c) $y[n] = x[n] - x[n-2]$

(d) $y[n] = x[n] * x[n-1]$

(e) $y[n] = \begin{cases} x[n] & n = \text{multiple of } 3 \\ 0 & n \neq \text{multiple of } 3 \end{cases}$

(f) $y[n] = x[2-n] + x[-2-n]$

(g) $y[n] = x^*[-n+2]$

Problem 127. Consider a real signal $x[n]$ with Fourier transform $X(e^{jw})$. Determine $y[n]$ for each of the following signals based on $x[n]$.

(a) $Y(e^{jw}) = Re\{X(e^{jw})\}$

(b) $Y(e^{jw}) = X(e^{jw}) + X(e^{-jw})$

(c) $Y(e^{jw}) = \frac{d}{dw}\{e^{jw}X(e^{jw})\}$

(d) $Y(e^{jw}) = \frac{d}{dw}\{e^{-jw}\left[X(e^{j(w+\frac{\pi}{5})}) - X(e^{j(w-\frac{\pi}{5})})\right]\}$

Problem 128. A causal discrete-time LTI system is described by

$$y[n] + \frac{1}{2}y[n-1] - \frac{3}{4}y[n-2] = 2x[n-2] + x[n-1] - \frac{5}{2}x[n]. \tag{7.1}$$

where $x[n]$ and $y[n]$ are the input and output of the system, respectively.

(a) Find the frequency response of the system.

(b) Determine the corresponding filter type.

Problem 129. Let $x[n]$ have the Fourier transform $X(e^{jw})$. Determine $x[n]$ for each of the following cases:

(a)

$$X(e^{jw}) = \begin{cases} 1 & \frac{\pi}{4} < |w| < \frac{3\pi}{4} \\ 0 & \text{otherwise} \end{cases} \tag{7.2}$$

(b) $x[n]$ is a real and even signal and

$$\mathcal{R}e\{X(e^{jw})\} = \frac{1}{1 - \frac{1}{2}e^{-jw}}. \tag{7.3}$$

Problem 130. Consider the following system with input $x[n]$ and output $y[n]$ as below. Determine whether each system can be LTI or not and if the system is LTI find the frequency response of the system.

(a) $x[n] = (\frac{1}{4})^n u[n], \qquad y[n] = 4(\frac{1}{3})^n u[n].$

(b) $x[n] = \frac{\sin(n\pi/8)}{n\pi}, \qquad y[n] = \frac{\sin(n\pi/4)}{n\pi}.$

Problem 131. Let $X(e^{jw})$ denote the Fourier transform of the signal $x[n]$ depicted in Fig. 7.3.

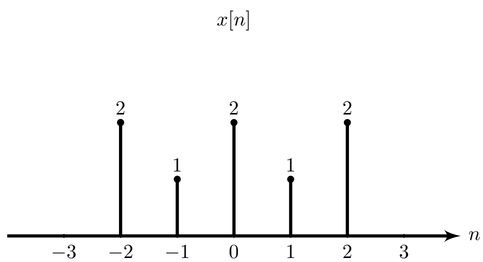


Figure 7.3: Problem 131

(a) Find $X(e^{j0})$.

(b) Find $X(e^{j\pi})$.

(c) Find $\angle X(e^{jw})$.

(d) Find $\int_{-\pi}^{\pi} X(e^{jw})dw$.

(e) Evaluate $\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw$.

(f) Evaluate $\int_{-\pi}^{\pi} |\frac{d}{dw} X(e^{jw})|^2 dw$.

Problem 132. Consider an LTI system with input $x[n]$ and output $y[n]$. Find the frequency response and the impulse response of the systems corresponding to each of the following cases.

(a) $x[n] = (\frac{1}{3})^n u[n], \qquad y[n] = \frac{1}{2}(\frac{1}{3})^n u[n] + (\frac{1}{4})^n u[n].$

(b) $x[n] = (\frac{1}{3})^n u[n], \qquad y[n] = (\frac{1}{3})^n u[n] + (\frac{1}{3})^{n-2} u[n-2].$

Problem 133. Consider the causal LTI system described by the following difference equation. Determine the difference equation that characterizes the inverse system, both directly and using the Fourier transform.

$$y[n] + \frac{1}{2} y[n-1] = x[n]. \tag{7.4}$$

Problem 134. For each of the following frequency responses, determine a difference equation that relates the output $y[n]$ to the input $x[n]$.

(a) $H(e^{jw}) = 1 + \frac{7e^{-jw}}{(1-e^{-jw})(3+e^{-jw})}$

(b) $H(e^{jw}) = \frac{e^{j2w}}{1+\cos(w)}$

Problem 135. Consider a discrete-time system with input $x[n]$ and output $y[n]$. The Fourier transforms of these signals are related through the following equation:

$$Y(e^{jw}) = X(e^{jw}) + 2e^{-2jw}X(e^{jw}) - \frac{d}{dw}X(e^{jw}). \tag{7.5}$$

(a) Is the system linear?

(b) Is the system time-invariant?

(c) Find the output of this system for the input $x[n] = \delta[n]$.

Problem 136. Consider a discrete-time signal $x[n]$ with Fourier transform as illustrated in Fig. 7.4 and let

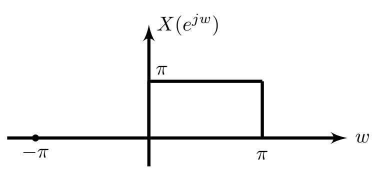


Figure 7.4: Problem 136.

$$A = \sum_{n=-\infty}^{+\infty} jIm\{x[n]\}e^{jn\pi/2}. \tag{7.6}$$

$$h(t) = \sum_{n=-\infty}^{+\infty} 2x[-n]e^{jnt}. \tag{7.7}$$

(a) Evaluate A.

(b) Evaluate $B = \int_0^{\pi/4} h(t)dt$.

Problem 137. Signal $x[n]$ has Fourier transform $X(e^{jw})$ as illustrated in Fig. 7.5. Obtain the following values:

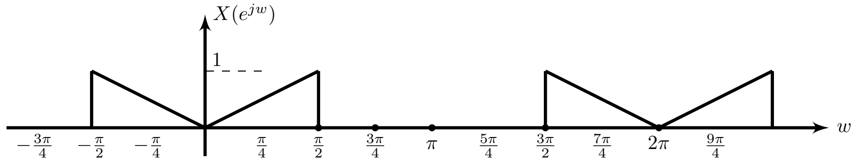


Figure 7.5: Problem 137.

(a) $x[0]$

(b) $\sum_{n=-\infty}^{+\infty} x[n]$

(c) $\sum_{n=-\infty}^{+\infty} |x[n]|^2$

Problem 138. Consider a system whose input $x[n]$ and output $y[n]$ are related with the following relationship:

$$y[n] = (-1)^n w[n] + w[n],$$

$$w[n] = x[n] * h[n],$$

where the frequency response of the system $H(e^{jw})$ is sketched in Fig. 7.6.

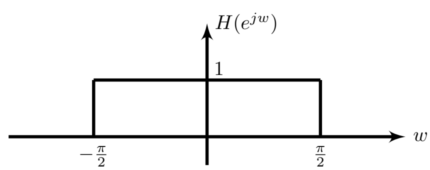


Figure 7.6: Problem 138, The frequency response $H(e^{jw})$.

Find the output of the system to the input

$$x[n] = \delta[n].$$

Problem 139. Consider a discrete-time system with a real impulse response $h[n]$. Let $g[n]$ be another system where $G(e^{jw}) = H(e^{jw})H(e^{-jw})$. Is $g[n]$ real?

Problem 140. Consider a discrete-time system shown in Fig. 7.7,

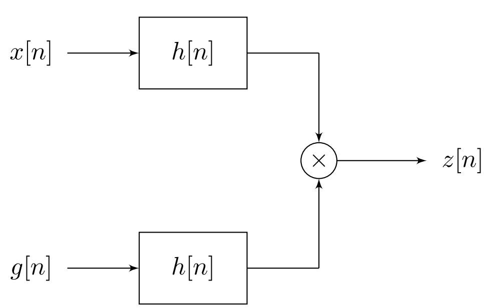


Figure 7.7: Problem 140

where

$$x[n] = 1 + \cos(\frac{\pi}{5}n) + \cos(\frac{3\pi}{5}n). \tag{7.8}$$

$$g[n] = 1 + 2\cos(\frac{2\pi}{5}n), \tag{7.9}$$

$$h[n] = \frac{2\sin(\frac{\pi}{4}n)}{n\pi}\cos(\frac{\pi}{3}n). \tag{7.10}$$

Find and sketch $Z(e^{jw})$ and $z[n]$.

Problem 141. Evaluate each of the following integrals and sums.

(a) $\int_{-\pi}^{\pi} \frac{e^{jw}}{1-0.5e^{-jw}} dw$

(b) $\sum_{n=-\infty}^{+\infty} \frac{\sin(n\frac{\pi}{4})}{n\pi} \cdot \frac{\sin(n\frac{\pi}{6})}{n\pi}$

(c) $\sum_{n=0}^{+\infty} n(\frac{1}{n})^n$

Problem 142. Signal $x[n]$ has Fourier transform $X(e^{jw})$ as illustrated in Fig. 7.8. Assume that

$$z[n] = x[n] \cdot y[n], \tag{7.11}$$

where

$$y[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k]. \tag{7.12}$$

Find $z[n]$ and its Fourier transform.

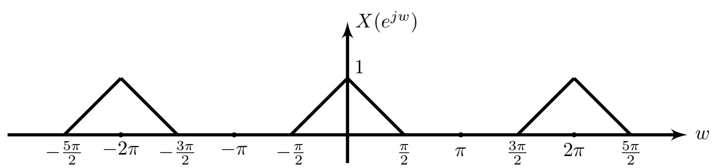


Figure 7.8: Problem 142

Problem 143. Consider an ideal low-pass filter with frequency response

$$H(e^{jw}) = \begin{cases} 1 & |w| < \frac{3\pi}{4} \\ 0 & |w| > \frac{3\pi}{4} \end{cases}.$$

The input to this filter is

$$x[n] = \sum_{k=-\infty}^{+\infty} \left[2(-1)^k\delta[n-2k] + \delta[n-2k]\right].$$

Find the output of this filter.

Problem 144. (a) A causal LTI system is described by the difference equation

$$y[n] - ay[n-1] = bx[n] + x[n-1], \tag{7.13}$$

where a is real and with magnitude less than 1.

Find the relation between the values of a and b such that the frequency response of the system satisfies

$$|H(e^{jw})| = 1, \quad \text{for all } w.$$

(b) Find and sketch the output of this system with $a = \frac{1}{3}$ when the input is

$$x[n] = (\frac{1}{3})^n u[n].$$

Use the value of b obtained in Part (a).

Problem 145. Let $x[n]$ be a signal with Fourier transform $X(e^{jw})$.

(a) If

$$X(f) = \prod(\frac{2\pi f}{5})$$

Find the value of

$$A = \sum_{n=-\infty}^{+\infty} n^2 x[n]. \tag{7.14}$$

(b) If

$$x[n] = \cos(\pi n)(u[n+1] - u[n-1]).$$

Find the value of

$$B = \int_{-\pi}^{\pi} |X(e^{jw})\cos(w)|^2 dw. \tag{7.15}$$

Problem 146. Consider an LTI system with input $x[n]$ and output $y[n]$. Let the input $x[n]$ be

$$x[n] = \cos(\frac{n\pi}{4}) + \cos(\frac{3n\pi}{5}). \tag{7.16}$$

Find the output of this system if the impulse response $h[n]$ is

$$h[n] = \frac{\sin(n\pi/2)}{n} * e^{jn\pi/2} \frac{\sin(n\pi/2)}{n}. \tag{7.17}$$

Problem 147. Find the inverse Fourier transform of

$$X(e^{jw}) = \Big[e^{-j10w}\frac{\sin(15w/2)}{\sin(w/2)}\Big] * \Big[\frac{d}{dw}(\frac{1}{1-\frac{1}{2}e^{-jw}} + 1)\Big].$$

Problem 148. Consider signals $x_1[n]$ and $x_2[n]$ below with Fourier transforms $X_1(e^{jw})$ and $X_2(e^{jw})$, respectively. Use the Fourier transform properties to determine whether the time-domain signals are complex, real and even, imaginary and odd.

$$X_1(e^{jw}) = e^{-jw}\cos(w)$$
$$X_2(e^{jw}) = j\sin(w)\cos(w).$$

Problem 149. Consider the signal depicted in Fig. 7.9. Let the Fourier transform of this signal be written as

$$X(e^{jw}) = A(w) + jB(w).$$

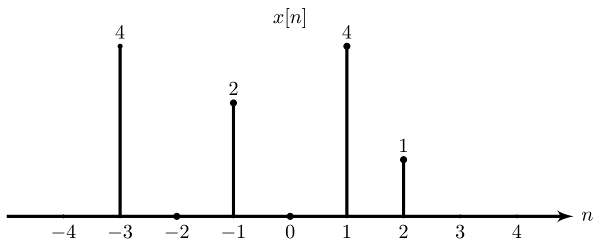


Figure 7.9: Problem 149, the input $x[n]$ to the system.

If the Fourier transform of $y[n]$ is presented as

$$Y(e^{jw}) = 2B(w)e^{jw} + jA(w),$$

find the value of $y[n]$ at $n = 1$.

Chapter 8

Sampling

Problem 150. Determine the minimum sampling frequency for the following signals to avoid aliasing.

(a) $x(t) = \left(\frac{\sin(4000\pi t)}{\pi t}\right)^2$

(b) $x(t) = \frac{2\sin(4t)\sin(8t)}{\pi t^2}$

Problem 151. Let $x(t)$ be a band-limited signal for which the corresponding Nyquist rate is w_0. Determine the Nyquist rate for each of the following signals.

(a) $x(2t)$

(b) $x(t) * x(t)$

(c) $x^2(2t)$

(d) $x(t) \cdot \cos(2w_0 t)$

(e) $\frac{dx(t)}{dt}$

Problem 152. For each of the following signals, determine the maximum sampling interval T such that the signals are recoverable from the sampled signals through the use of an ideal low-pass filter.

(a) $x(t) = \frac{\sin(3\pi t)}{t} + \cos(2\pi t)$

(b) $x(t) = \frac{\sin(\pi t)}{2t} \cdot \cos(2\pi t)$

(c) $x(t) = e^{-3t}u(t) * \frac{\sin(Wt)}{\pi t}$

Problem 153. Shown in Fig. 8.1, is a system in which the input signal is multiplied by a periodic square wave. The period of $s(t)$ is 2π. The input signal is band-limited with $|X(jw)| = 0$ for $|w| \geq w_m$. The sampler is followed by an ideal low-pass filter with gain 1 and cutoff frequency w_c, for reconstruction of $x(t)$ from its samples $x_s(t)$. Determine the maximum value of w_m.

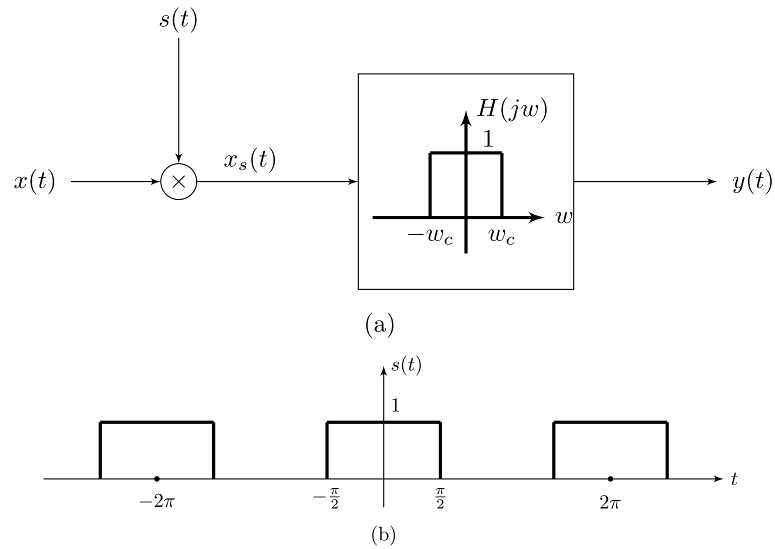


Figure 8.1: Problem 153

Problem 154. In the system shown in Fig. 8.2, the signal $x[n]$ is the input to the system S_1. The system S_1 is defined by

$$S_1 : w[n] = x[2n].$$

The output $w[n]$ of the system S_1 is sampled by a periodic impulse train. The signal $x[n]$ is band limited to $w = \frac{\pi}{4}$; that is,

$$X(e^{jw}) = 0, \ |w| \geq \frac{\pi}{4}.$$

Determine the maximum value of N such that $w[n]$ is recoverable from $w_p[n]$ through the use of an ideal low-pass filter.

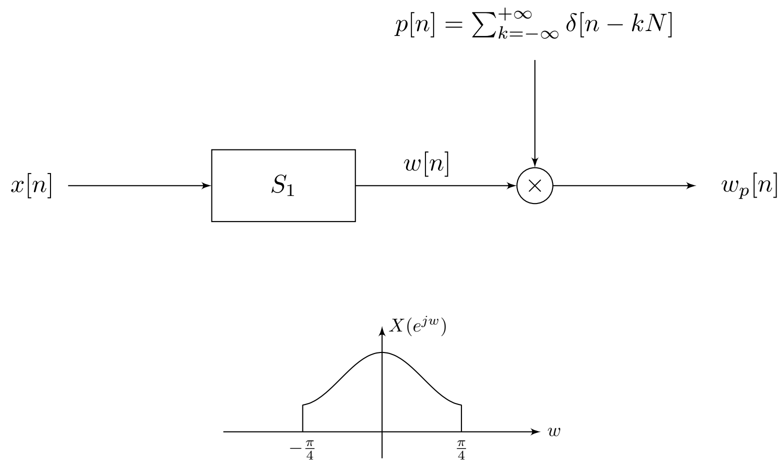


Figure 8.2: Problem 154

Problem 155. Consider the system shown in Fig. 8.3. The system consisting of a casual continuous-time LTI system followed by a sampler, Continuous-to-Discrete (C/D) Converter, and an LTI discrete-time system. The relationship between $x(t)$ and $y(t)$ is given by:

$$\frac{dy(t)}{dt} + 2y(t) = x(t). \tag{8.1}$$

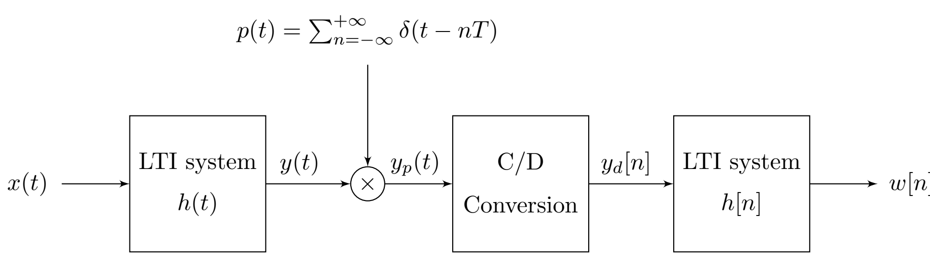


Figure 8.3: Problem 155

Determine $h[n]$ such that if the input of the system is $x(t) = \delta(t)$, then the output is equal to $w[n] = \delta[n]$.

Problem 156. Consider the continuous-time filter with the frequency response as follows:

$$H(jw) = \frac{1}{jw + 1}.$$

We want to implement the discrete-time version of this filter by setting $h[n] = h(nT)$.

Determine the value of T such that the half power bandwidth (3dB bandwidth) is equal to $w = \frac{\pi}{2}$.

Problem 157. Let $x(t)$ be a continuous-time signal whose Fourier transform has the property that $X(jw) = 0$ for $|w| > w_0$. Signal $x(t)$ is sampled at sampling frequency $w_s > 2w_0$, to obtain signal $x_d[n]$.

Determine the relationship between the energy E_d of $x_d[n]$ and the energy E of the original signal, if the sampling period is T. Note that the energy in a continuous-time function $x(t)$ is defined as

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt,$$

and the energy of a sequence $x[n]$ is defined as

$$E_d = \sum_{n=-\infty}^{+\infty} |x[n]|^2.$$

Problem 158. Shown in Fig. 8.4(a), is a system that processes continuous-time signals using a discrete-time filter $h[n]$. The Fourier transform of the input signal is as indicated in Fig. 8.4 (b).

$$h[n] = \delta[n],$$

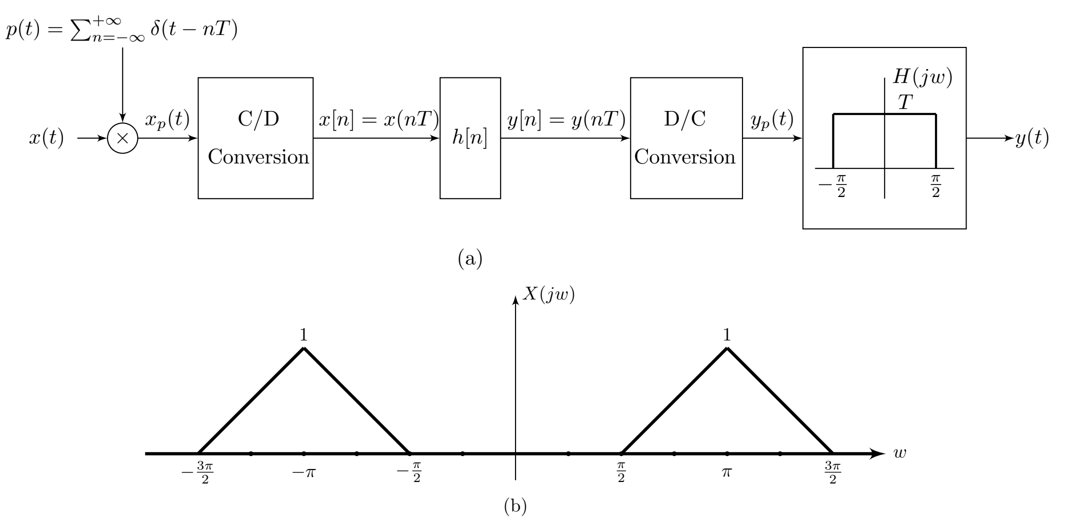


Figure 8.4: Problem 158

(a) Determine the maximum sampling interval T such that $x(t)$ is recoverable from $x[n]$ through the use of a low-pass filter.

(b) If $T = 2$, sketch $X_p(jw)$.

(c) If $T = 2$, sketch $Y(e^{jw})$ and $Y(jw)$.

Chapter 9

Laplace Transform

Problem 159. Use the definition to obtain the Laplace transform of the following signals:

(a) $x(t) = 3e^{2t}u(t)$

(b) $x(t) = 3e^{-2t}u(t) + te^{-t}u(t)$

(c) $x(t) = e^{-2|t|}$

(d) $x(t) = u(t+2)$

Problem 160. For each of the following signals, use Laplace transform properties to determine Laplace transforms.

(a) $x(t) = te^{-t}u(t)$

(b) $x(t) = te^{-t}\cosh(t)u(t)$

(c) $x(t) = \int_0^t e^{-2\tau}\cos(\tau)d\tau$

(d) $x(t) = \int_0^t (t-\lambda)^4 \sin(\lambda)d\lambda$

(e) $x(t) = \sin^2(wt)u(t)$

(f) $x(t) = te^{-2|t|}$

(g) $x(t) = \cos(t)u(t) + e^{-t}u(-t)$

Problem 161. For each of the following signals find the Laplace transform.

(a) $x(t)$ as in the figure.

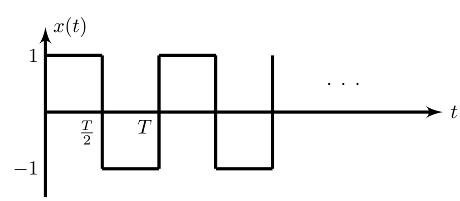


Figure 9.1: Problem 161 - Part (a)

.

(b) $x(t)$ as in the figure.

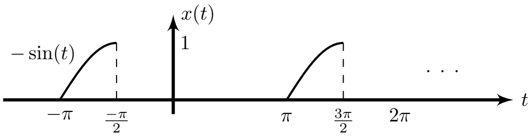


Figure 9.2: Problem 161 - Part (b)

.

(c) $x(t)$ as in the figure.

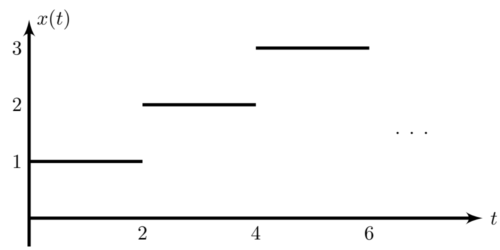


Figure 9.3: Problem 161 - Part (c)

Problem 162. Let $\mathcal{L}\{x(t)\} = X(s)$. For each of the following signals find the Laplace transforms based on $X(s)$.

(a) $y(t) = x(3t)$

(b) $y(t) = x(t-2)$

(c) $y(t) = x(t) * \frac{d}{dt}x(t)$

(d) $y(t) = e^{-t}x(t)$

(e) $y(t) = 2tx(t)$

(f) $y(t) = \int_{-\infty}^{t} x(3\tau)d\tau$

(g) $y(t) = x^*(-2t+1)$

(h) $y(t) = x(t-3) + x^*(-t+2)$

(i) $y(t) = x(-3t)e^{(1+3j)t}$,
$-1 < Re(s) < 2$

Problem 163. For signals in Parts (a) and (b), determine the region of convergence for each of the following cases:

(1) If $x(t)$ is a right-sided signal.

(2) If $x(t)$ is a left-sided signal.

(3) If $x(t)$ is a two-sided signal and $\int_{-\infty}^{\infty} x(t)e^{-1.5t}dt = \infty$.

(a) $x(t)$ is a signal with Laplace transform $X(s) = \frac{s^2-4}{(s+1)(s^2-9)}$.

(b) $x(t)$ is a signal with Laplace transform $X(s) = \frac{1-e^s}{(s+2)(s+1)}$.

Problem 164. Determine the inverse Laplace transform for each of the following cases.

(a) $F(s) = \frac{2(s+6)}{(s+1)(s^2+5s+6)}$

(b) $F(s) = \frac{100(s+4)}{s^2(s^2+2s+10)}$

(c) $F(s) = \frac{4s^2}{(s^2+2s+3)(s^2+1)}$

(d) $F(s) = \frac{5}{(s^2+6s+10)(s+1)}$

(e) $F(s) = \frac{2s^3+8s^2+11s+3}{(s+2)(s+1)^3}$

Problem 165. Find the values of $f(0)$ and $f(\infty)$ for the following Laplace transforms using the initial and final value theorems.

(a) $F(s) = \frac{10s-2}{s^2+6s+10}$

(b) $F(s) = \frac{2s^3-s^2-3s-10}{s^3+3s^2+6s}$

(c) $F(s) = \frac{10s-2}{s^2-6s+10}$

(d) $F(s) = \frac{10s^2-2}{(s+2)^2(s+1)(s^2+5s+10)}$

Problem 166. **(a)** Let $y(t)$ be the output of a causal LTI system with transfer function $H(s) = \frac{3}{s-3}$ when the input is $x(t) = u(t)$. Find the value $y(t)$ when t tends to infinity.

(b) Solve the problem when the system is stable instead of being causal.

Problem 167. Using Laplace transform, determine which of the following systems are LTI.

(a) $x(t) = e^t u(t), \qquad y(t) = -e^{2t}u(-t)$

(b) $x(t) = e^{3t} u(t), \qquad y(t) = -e^{t}u(-t)$

Problem 168. Find the inverse Laplace transform of following functions by considering their region of convergence.

(a) $X(s) = \dfrac{s+1}{(s+1)^2+9} - \dfrac{5}{s+2} - \dfrac{1}{s-2}, \qquad -1 < Re\{s\} < 2$

(b) $X(s) = \dfrac{s+2}{s^2+7s+12}, \qquad -4 < Re\{s\} < -3$

(c) $X(s) = \dfrac{1-e^{-s}}{s(1-e^{-3s})}, \qquad Re\{s\} > 0$

(d) $X(s) = \dfrac{1}{s^2(s+1)^2}, \qquad -1 < Re\{s\} < 0$

(e) $X(s) = \dfrac{s-5}{s^2-6s+10}, \qquad Re\{s\} > 3$

Problem 169. Consider a stable and LTI system which it is described with the following differential equation. Which of the following inputs: $x_1(t) = e^{-t}, x_2(t) = e^{-2t}, x_3(t) = e^{8t}$, could result in a bounded output?

$$9y(t) - \frac{d^2y(t)}{dt^2} = x(t) \tag{9.1}$$

Problem 170. Transfer function of an LTI system is $H(s) = \frac{4s}{s^2+s-6}$, $-3 < Re\{s\} < 2$. Determine the correctness of the following propositions:

(a) System is stable.

(b) System is non-causal.

(c) If $h(t)$ is the impulse response of the system, then $\int_{-\infty}^{\infty} h(t)dt = 0$.

Problem 171. Let $X(s)$ be the Laplace transform of $x(t)$. The following information is available on $X(s)$.

(1) $X(0) = 4$

(2) $X(s)$ has a pole in $s = -1 + j$

(3) $s = 3$ is not in the ROC.

What can be said about the ROC of $X(s)$?

Problem 172. Transfer function of an LTI system is $H(s) = \frac{s+3}{s^2-4s+4-\alpha^2}$. For all values of α, investigate the causality and stability of this system.

Problem 173. The Laplace transform of $x(t)$ has the following pole-zero diagram. If the region of convergence is $-1 < Re\{s\} < 1$, find $x(t)$.

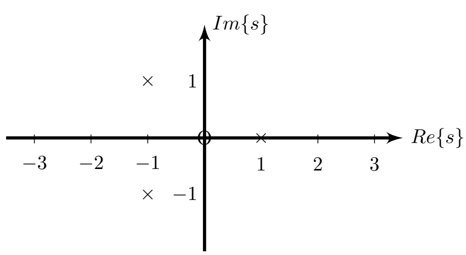


Figure 9.4: Problem 173 - The location of poles and zeros

Problem 174. Consider an LTI system whose response to the input

$$x(t) = te^{-3t}u(t)$$

is the output below:

$$y(t) = e^{-2t}[\cos(t) + \sin(t)]u(t) - e^{-3t}u(t).$$

(a) Find the system transfer function and its region of convergence. Is the system causal? Is the system stable?

(b) Find the differential equation corresponding to this system.

Problem 175. Evaluate the following integrals using the Laplace transform properties and pairs in Tables **??** and **??** and in particular the Laplace-domain integration property.

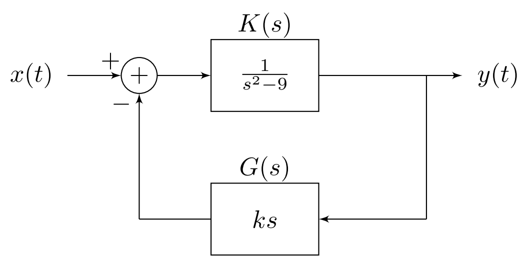
(a) $\int_0^\infty \frac{\sin(t)}{t} dt$

(b) $\int_0^\infty te^{-3t}\cos(t)dt$

(c) $\int_0^\infty \frac{\cos(2t)-\cos(3t)}{t} dt$

(d) $x(t) = e^{-2|t|} \Rightarrow \begin{cases} A = \int_{-\infty}^{\infty} x(t)e^{t}dt \\ B = \int_{-\infty}^{\infty} x(t)e^{-5t}dt \end{cases}$

Problem 176. Consider the following causal system:



(a) Find the transfer function of the system.

(b) For $k = 0$, find the impulse response of the system, and determine if the system is stable or not.

(c) Find the value of k for which the system has two poles at $s = 1$ and $s = -9$.

Problem 177. The Laplace transform of the impulse response for a system is given as follows:

$$H(s) = \frac{1}{(s+2)(s-3)}.$$

(a) Find the region of convergence for the transfer function for which the system is stable and causal.

(b) Find the impulse response assuming the system is stable.

(c) Find the impulse response when the system is neither causal nor stable.

Problem 178. The following information about an LTI system with input $x(t)$ and output $y(t)$ is at hand:

$$X(s) = \frac{s+2}{s+3} \quad \text{and} \quad x(t) = 0 \text{ for } \quad t < 0$$

and

$$y(t) = \frac{1}{5}e^{-3t}u(t) - \frac{4}{5}e^{2t}u(-t).$$

Find the response of the system to the input $x_1(t) = e^{-2t}$ and $x_2(t) = e^{3t}$.

Problem 179. Step response of an LTI system is $s(t) = (1 - e^{-2t} - 2te^{-2t})u(t)$. If the system response to the input $x(t)$ is

$$y(t) = (1 + e^{-4t} - 2e^{-2t})u(t),$$

find $x(t)$.

Problem 180. If an LTI system where transfer function $H(s)$ is invertible, the transfer function of the inverse system, $H_i(s)$, is given by $H_i(s) = \frac{1}{H(s)}$. Using this fact, work out the following questions.

(a) The impulse response for systems S_1 is as below. Investigate reversibility of this system.

$$h_1(t) = e^{-t}u(t).$$

(b) The pole-zero diagram of an LTI system, and its ROC is indicated in the following figure. Find the region of convergence of the inverse system $H_i(s) = \frac{1}{H(s)}$.

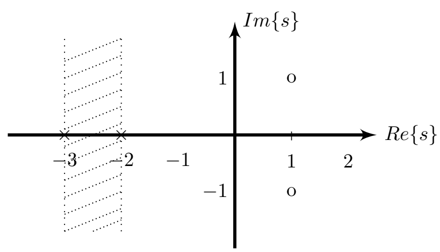


Figure 9.5: Problem 180 - Part (b), The pole-zero diagram of $H(s)$

Problem 181. There is an LTI system with the input and output as below. Find the impulse response of this system.

$$x(t) = 2e^{-4t}u(t) + 2e^{-2t}u(-t),$$
$$y(t) = e^{-4t}u(t) - e^{-3t}u(t).$$

Chapter 10

Z-Transform

Problem 182. For each of the following signals find the z-transform.

(a) $x[n] = \cos(nw_0)u[n]$

(b) $x[n] = -a^n u[-n-1]$

(c) $x[n] = (\frac{1}{2})^n u[3-n]$

(d) $x[n] = a^{|n|}, \quad |a| < 1$

(e) $x[n] = (\frac{1}{4})^{n-1} u[n-1] - 3(\frac{1}{2})^n u[-n-1]$

Problem 183. Determine the z-transform for each of following signals using z-transform properties.

(a) $x[n] = (2)^n \cos(\frac{n\pi}{3})u[-n-1]$

(b) $x[n] = n(\frac{1}{2})^n u[n-2]$

(c) $x[n] = \frac{1}{n}(-2)^n u[-n-1]$

Problem 184. For each of the following signals, find the z-transform using convolution and differentiation properties.

(a) $x[n] = n\big[(\frac{1}{2})^n u[n] * (\frac{1}{3})^n u[n-2]\big]$

(b) $x[n] = n\sin(\frac{\pi}{2}n)u[-n]$

(c) $x[n] = (\frac{1}{2})^{|n|}\big[u[n+3] - u[n-4]\big]$

(d) $x[n] = \frac{1}{n}u[n]$

Problem 185. For each of the following signals find the z-transform based on the z-transform of $x[n]$.

(a) $y[n] = x_{(4)}[n-2]$

(b) $y[n] = e^{-jn}x^*[n]$

(c) $y[n] = \begin{cases} -x[n] & \text{even } n \\ 2x[n] & \text{odd } n \end{cases}$

Problem 186. The transfer function of a causal LTI system is as follows

$$H(z) = \frac{1-5z^{-1}}{1+\frac{1}{2}z^{-1}}.$$

Find the system response to the input

$$x[n] = \begin{cases} 1+5^n & n \geq 0 \\ 1 & n < 0 \end{cases}.$$

Problem 187. Consider an LTI, causal and stable system with the impulse response $h[n]$. If the z-transform of the system output to the input $x[n] = h[-n]$ is

$$Y(z) = \frac{25z}{(5z-1)(5-z)},$$

find the value of following expressions.

(a) $A = \sum_{n=0}^{n=\infty} h[n]$

(b) $B = \sum_{n=0}^{n=\infty} h^2[n]$

Problem 188. For a causal LTI system we know that when the input is $x[n] = (\frac{1}{2})^n u[n]$, for the output $y[n]$ we have

$$y[0] = 2,\ y[1] = 4,\ y[2] = -3,\ y[3] = 1.$$

Find the value of $h[3]$.

Problem 189. The transfer function of an LTI system with the impulse response $h[n]$ is described by

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{z^{-1}(1-\frac{1}{3}z^{-1})(1+3z^{-1})}, \quad ROC: |z| > 3.$$

Comment on the following items:

(a) Causality

(b) Stability

(c) The value of $\sum_{n=-\infty}^{\infty} h[n]$

Problem 190. Consider an LTI and causal system with the impulse response $h[n]$. The step response of this system is $s[n] = \delta[n] + ah[n-1]$, where a is a constant and positive number. Find $h[0]$ for different values of a.

Problem 191. Consider the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] + \frac{1}{18}y[n-2] = 2x[n].$$

(a) If the system is stable, find the response to the input $x[n] = \delta[n-1]$.

(b) Find the system response to $x[n] = e^{j\pi n}$.

Problem 192. The z-transform of signal $x[n]$ is $X(z) = \frac{z^2}{z^2-4}$. Find the z-transform of following signals.

(a) $y[n] = x[n-2]$

(b) $y[n] = (\frac{1}{2})^n x[n]$

(c) $y[n] = x[-n] * x[n]$

(d) $y[n] = nx[n]$

(e) $y[n] = \cos(2n)x[n]$

Problem 193. Find the inverse z-transform of each following signals using the partial fraction decomposition.

(a) $X(z) = \dfrac{4}{z^3(z+0.2)}, \quad |z| > \frac{1}{5}$

(b) $X(z) = \dfrac{4 - \frac{7}{4}z^{-1} + \frac{1}{4}z^{-2}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}}, \quad |z| > \frac{1}{2}$

(c) $X(z) = \dfrac{3 - 3z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})}, \quad x[n]$ is causal

(d) $X(z) = \dfrac{1}{1 + 3z^{-1} + 2z^{-2}}, \quad |z| > 2$

(e) $X(z) = \dfrac{1}{(1 - z^{-1})(1 - z^{-2})}, \quad |z| > 1$

Problem 194. Let $x[n] = n^2 5^n u[n]$ with $X(z)$ as its z-transform. Find $y[n]$ based on $x[n]$ for each of the following cases.

(a) $Y(z) = X(2z)$

(b) $Y(z) = X(z^{-1})$

(c) $Y(z) = \dfrac{dX(z)}{dz}$

(d) $Y(z) = \dfrac{z^2 - z^{-2}}{2} X(z)$

(e) $Y(z) = X(z)^2$

Problem 195. The transfer function of an LTI and causal system is as below

$$H(z) = \frac{1 + 2z^{-1}}{1 - \frac{1}{2}z^{-1}}.$$

Find an input signal which results the following output:

$$y[n] = \frac{1}{3}(\frac{1}{3})^n u[n] - 3^n u[-n-1].$$

Problem 196. Consider two causal systems described by the following difference equations. Let the input signal to these systems be $x[n] = (\frac{1}{2})^n u[n]$. Find the output for each system.

a) $y[n] - y[n-1] + y[n-2] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1]$.

b) $y[n] + 3y[n-1] = x[n] - x[n-1]$.

Problem 197. The impulse response of an LTI system is as below

$$h[n] = 2\delta[n] + \frac{5}{2}(\frac{1}{2})^n u[n] - \frac{7}{2}(-\frac{1}{4})^n u[n].$$

Is the inverse of the system causal and stable?

Problem 198. The following information is available for a system with the impulse response $h[n]$ and transfer function $H(z)$:

1. $h[n]$ is real and is equal to 0 for $n < 0$ and $n > 5$.
2. $\sum_{n=0}^{5} (-1)^n h[n] = 0$.
3. $H(z) = 0$ at $z = \frac{1}{2}e^{\frac{-j\pi}{4}}$, $z = 2e^{\frac{-j\pi}{4}}$.
4. $h[0] = 2$.
5. $H(e^{jw})$ has a pole at the origin.

Find $H(z)$.

Problem 199. Consider an LTI system for which the input $x[n]$ results in the output $y[n]$, both are shown in figures below. Investigate the causality and stability of this system.

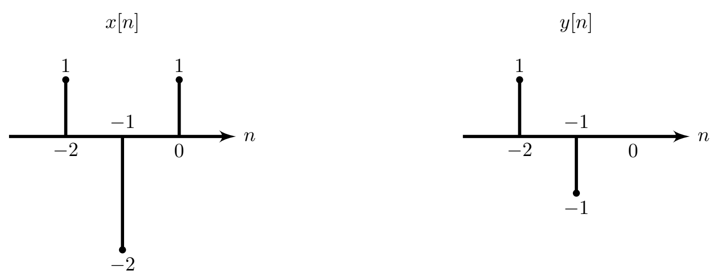


Figure 10.1: Problem 199

Problem 200. Consider two systems presented by the following equations. Let the impulse response of the inverse systems be denoted by $h_1[n]$ and $h_2[n]$, respectively. We let $h_3[n] = h_1[n] * h_2[n]$. Find $h_3[0]$.

$$y_1[n] = x[n] - \frac{1}{5}x[n-5],$$

$$y_2[n] = x[n] - \frac{1}{3}x[n-4].$$

Problem 201. Transfer function of a system is given as

$$H(z) = \frac{1}{1 - \frac{1}{2}e^{\frac{j\pi}{4}}z^{-1}} + \frac{1}{1 - \frac{1}{2}e^{\frac{-j\pi}{4}}z^{-1}} - \frac{1}{1 + 2z^{-1}}.$$

Obtain the impulse response $h[n]$ assuming that the system is

a) Stable

b) Causal

Problem 202. Find the z-transform of the following function when $|a| < 1$.

$$y[n] = \begin{cases} \sum_{k=-n}^{k=n} a^{|k|} & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Problem 203. Consider an LTI and causal system which has only one pole and only one zero. In addition, the impulse response satisfies the following conditions. Find the zero and pole of this system.

a) $h[0] = \frac{1}{2}$

b) $\sum_{n=-\infty}^{n=\infty} h[n] = 2$

c) $\sum_{n=-\infty}^{n=\infty} (-1)^n h[n] = 0$

Problem 204. Transfer function of an LTI system is assumed as

$$H(z) = \frac{2}{1 - \frac{1}{2}z^{-3}}, \quad |z| > \frac{1}{2}.$$

Find the response of the system to the input $x[n] = 2 + \cos(\frac{n\pi}{3})$.

Problem 205. Let the z-transform of $x[n]$ be as follows:

$$X(z) = \frac{1}{1 + z^2}, \quad ROC : |z| < 1.$$

Find the values of $x[0]$ and $x[-2]$.

Problem 206. Consider the following difference equation which describes a stable LTI system:

$$y[n] - ay[n-1] + y[n-2] = x[n]. \tag{10.1}$$

Comment on the causality of this system for different values of a.